

Example of the mathematics examination

Choose one of the two algebra problems, and one of the two analysis problems, and answer them.

Algebra 1

Suppose that x and a are an arbitrary vector and a constant vector in a vector space, V , respectively. Show that the mapping f that transforms x to $x + a$ is not a linear mapping.

Algebra 2

Suppose that

$$A = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}, \text{ and } N = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

(1) Show that $AN = NA$.

(2) Calculate N^2 and N^3 .

(3) Calculate $(A + N)^k$

Analysis 1

Show the value of the following equation:

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

Analysis 2

Solve $f(x)$ for the following differential equation,

$$\frac{d^2 f(x)}{dx^2} + 2 \frac{df(x)}{dx} - 3f(x) = 0$$

subject to $f(0) = 0$ and $f'(0) = 4$.