Self-organizing Network Learning of Sub-millisecond Temporal Coded Information

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Abstract

In this article, we report a simulation result of unsupervised learning characterized as temporally and spatially local. After the learning, the network preserves an input sequence whose intervals vary in sub-millisecond order, which is much smaller than the spike emission interval of the neurons. This is achieved by the forming of systematic local structures called *fractions*, namely, by selecting appropriate connections from the input neurons and other processing neurons. As a result, the network successfully becomes a sub-millisecond temporal information processing system in a self-organizing manner.

1 Introduction

A temporal structure of a neuronal activity has been considered to play an important role in defining the function of neuronal assemblies. It has also been considered that the behavior of a single neuron is highly dependent on the timing of its input spikes [1, 2]. According to Softky and Koch, a careful observation of the inter-spike interval suggests that cortical neurons behave more like a coincidence detector of incident spikes, rather than an integrator [3, 4]. From this point of view, hypothetical models have been developed for a single neuron.

Synaptic changes, i.e., LTP or LTD, are thought to correlate with the timing of input spikes to the synapse [5, 6]. Gerstner et al. devised a time-dependent learning rule for synaptic plasticity, and gave a solution for the longtime problem of how barn owls detect a sound location, i.e., sub-millisecond temporal information. They employ a strongly-leaky integrate-and-fire neuron model. The time-dependent learning rule chooses appropriate connections to the neuron from the input layer. Consequently, the model is able to distinguish the input timing in sub-millisecond order, which is much smaller than the spike

emission interval of the neuron [7]. However, they do not consider a network of such neurons or network learning.

Our standpoint is that the main term for conveying information is the network structure, which preserves the transmission delay between neurons, rather than the inter-spike interval of each neuron. From this point of view, the local synaptic learning process can be considered as a local structure change for creating a globally meaningful network structure.

2 Simulation model

This simulation model consists of a processing layer and an input layer. The processing layer is composed of 16 processing neurons. The input layer is composed of 16 input neurons.

2.1 Input layer

The j-th input neuron emits spikes to the i-th processing neuron, according to the probability:

$$P_j(t) = \frac{\Delta t}{\sigma\sqrt{2\pi}} \sum_{m=0}^{\infty} \exp\left[-\frac{(t - T_m^j - \Delta_{ij})^2}{2\sigma^2}\right]. \quad (1)$$

Every processing neuron is connected to every input neuron, and the transmission delay from the j-th input neuron to the i-th processing neuron, Δ_{ij}, obeys $N(1.00, 0.01)$.[1] T_m^j is the m-th spike time of the j-th input neuron [Fig.1]. σ is the input jitter 0.02[ms], and Δt is the time step. Every processing neuron also receives white-noise spikes, which are produces by 10 additional input neurons. They fire at the rate of one spike per millisecond.

2.2 Processing layer

The processing layer is a one-dimensional network with a periodical boundary condition. Neurons are indexed by their spatial location. Neuron i is bidirectionally connected to the neighboring neurons $i-5, ..., i-1, i+1, ..., i+5$. Each pair of connected neurons has four synaptic connections. A spike from the i-th pre-synaptic neuron is transmitted through the axon to the j-th post-synaptic neuron with an axonal delay time of $\Delta_{ij}^{\mathrm{ax}} = 0.25 \cdot |i - j|$[ms]. The spike provokes the synapses to release neurotransmitters, which evoke an excitatory post-synaptic potential (EPSP). The time delay of the evoked EPSP via the qth synapse, $\Delta_{ij}^{\mathrm{syn}\ q}$, obeys $N(0.9, 0.18)$. Then, the transmission delay from the i-th pre-neuron to the j-th post-neuron via the q-th synapse is defined by $\Delta_{ij}^q = \Delta_{ij}^{\mathrm{ax}} + \Delta_{ij}^{\mathrm{syn}\ q}$. The transmission delay mainly depends on the synaptic delay. Although the real axonal delays are small enough to be ignored, we set them to be relatively large, in order to discriminate the neuronal connections and to easily see the result.

[1] In this article, $N(a, b)$ denotes the Gaussian distribution whose mean and standard deviation are a[ms] and b[ms], respectively.

2.3 Neuron model

The membrane potential of the i-th neuron is defined as $v_i = h_i + h_i^{ref}$, where h_i and h_i^{ref} represent an integration of EPSPs and a refractory potential, respectively. When the membrane potential v_i exceeds the threshold θ, an action potential is provoked, and v_i is reset to the resting potential. h_i is given by [7, 8]:

$$\frac{dh_i}{dt} = -\frac{h_i}{\tau_m} + \sum_{j,f,q} \frac{J_{ij}^q}{\tau_s} \exp\left[-\frac{t - t_{jq}^f}{\tau_s}\right]. \tag{2}$$

t_{jq}^f is the evoked time of the f-th EPSP in the i-th neuron by the j-th neuron through the q-th synapse, and J_{ij}^q is the transmission efficacy of the synapse. If the neuron j fires at t_j', $t_{jq}^f = t_j' + \Delta_{ij}^q$. The membrane time constant τ_m and the rise time of EPSP τ_s are 0.1[ms], and then the half-width of EPSPs with these values is 0.25[ms]. This width seems narrow from the known physiological data. According to Softly and Koch, however, cortical neurons should be described in a smaller constant $\tau_m \leq 0.2$[ms] than the real value in order to observe the plausible spike interval [3, 7].

After a spike, refractory currents due to a hyperpolarization occur. We define the refractoriness in our model as [3, 9] :

$$h_i^{ref}(t) = \sum_{n=1}^{N} \int_0^t d\tau\, \eta(\tau)\, \delta(t - \tau - t_i^n), \tag{3}$$

where t_i^n is the n-th spike time of the neuron i. Refractory function $\eta(\tau)$ is defined by :

$$\eta(\tau) = \begin{cases} -\infty & (\text{for } 0 < \tau \leq \tau_{ref}) \\ -\frac{\eta_0}{\tau - \tau_{ref}} & (\text{for } \tau_{ref} < \tau\ ). \end{cases} \tag{4}$$

The upper and lower definitions correspond to the absolute and relative refractoriness, respectively.

An action potential not only propagates down to the axon, but also propagates backward to the dendrite [6]. After the i-th neuron fires at time t_i', the n-th "back-propagated" potential arrives at the synapse q, which is connected to the i-th neuron, at time $t_{iq}^n = t_i' + \Delta_i^{dend\,q}$. $\Delta_i^{dend\,q}$ obeys $N(0.05, 0.025)$.

3 Time-dependent learning rule

We assume that the learning process is localized spatially and temporally, and is defined by the timing of the pre-input and post-back-propagated action potentials [7, 6]. Here, the locality of the learning process is emphasized. Namely, each synaptic efficacy changes using only the current synaptic state and also on whether there is a pre-input or post-back-propagated spike at the present time.

The time-dependent learning rule introduced by Gerstner et al. [7] is given by

$$J_{ij}^q = J_0 + \sum_{f=1}^{F} [\gamma + \sum_{n=1}^{N} W(t_{jq}^f - t_{iq}^n)]. \tag{5}$$

J_0 is the initial value of the efficacy. $W(t)$ is an anti-symmetrical learning window:

$$W(t) = \begin{cases} 0.3 \exp\left[\frac{t}{0.5}\right] & (t \leq 0) \\ 0.5 \exp\left[-\frac{t}{0.5}\right] - 0.2 \exp\left[-\frac{t}{5}\right] & (t > 0) \ . \end{cases} \tag{6}$$

Namely, the synaptic plasticity depends on the timing between pre-input time t_{jq}^f and post-back-propagation time t_{iq}^n.

Alternatively, we employ the following learning rule. First, for a synapse q between two neurons j and i, we define three internal state variables α_1^{ijq}, α_2^{ijq}, and β^{ijq}, which exponentially decay by themselves with time.

$$\alpha_1^{ijq}(t + \Delta t) = \alpha_1^{ijq}(t) \cdot \exp\left[-\frac{\Delta t}{0.5}\right] \tag{7a}$$

$$\alpha_2^{ijq}(t + \Delta t) = \alpha_2^{ijq}(t) \cdot \exp\left[-\frac{\Delta t}{5}\right] \tag{7b}$$

$$\beta^{ijq}(t + \Delta t) = \beta^{ijq}(t) \cdot \exp\left[-\frac{\Delta t}{0.5}\right] . \tag{7c}$$

When a spike from the j-th pre-neuron evokes an EPSP at the i-th post-neuron via synapse q, the following updating takes place.

$$\Delta\beta^{ijq} = 0.3, \quad \Delta J_{ij}^q = \gamma + \alpha_1^{ijq}(t) - \alpha_2^{ijq}(t). \tag{8}$$

When an action potential from the i-th post-neuron back-propagates to the synapse q, which is connected to the j-th pre-neuron, the following updating takes place.

$$\Delta\alpha_1^{ijq} = 0.5, \quad \Delta\alpha_2^{ijq} = 0.2, \quad \Delta J_{ij} = \beta^{ijq}(t). \tag{9}$$

We can prove that learning rules (7), (8), and (9) are equivalent to (5) and (6). Although learning rules (5) and (6) do not explicitly satisfy the time locality, our reduced learning rules satisfy the time locality.

4 Simulation results

Let us show simulation results. Figure 2 shows the time-series of a neuron's membrane potential before learning. Figure 3 is a spike transmission diagram before learning. In Fig.3, each line connects the firing time of a pre-neuron to the time the EPSP is evoked on a post-neuron. [2] The ordinate denotes the neuron index, and the abscissa denotes the time. We can see that spikes are generated almost at random.

Figure 4 shows the learning process, where each line represents the synaptic efficacy; some of the synapses are enhanced while others are suppressed.[3] We can see that the contributing synapses to a neuron's firing change in time. For example, the connections with the white-noise input neurons are strengthened at the early learning stage. In the later stage, they are weakened, while

[2] For observation simplicity, the connections across the network boundary, like between the 0-th pre-neuron and the 15-th post-neuron, are omitted.

[3] Note that there are three kinds of connections. Initially, the connections from the white-noise input neurons have the largest efficacy. The connections from the processing neurons have the smallest efficacy.

the connections with the processing neurons are strengthened. Inputs are also selected. Namely, the connections with the input neurons, which have an appropriate input timing, remain.

Figure 5 shows the time-series of a neuron's membrane potential after learning. Spikes are generated at a certain timing. Figure 6 shows a spike raster after learning. [4] The spike timing of the neurons is almost regular, and highly correlated with the input timing. The relation between the firing time and the "evoked" time after learning is shown in Figure 7, where we can observe systematic structures called *fractions*. These *fractions* cooperatively preserve the input information, and enable the network to regularly produce spike trains. Since the *fractions* overlap with each other, the neurons seem to play several different roles. Consequently, the self-organized network structure becomes a distributed representation of the input sequence conveying sub-millisecond information.

When an adverse input sequence is provided to the trained network, the regular spike trains disappear [Fig.8]. This result indicates that the self-organized network structure is able to discriminate the learned input sequence from the adverse input sequence. Note that this difference is of sub-millisecond order.

5 Conclusion

In this article, we showed a simulation result of the unsupervised network learning characterized as temporally and spatially local. The network is able to preserve and discriminate the input sequence conveying sub-millisecond temporal information, although each neuron emits spikes at millisecond-order intervals. This is achieved by the forming of *fractions*, namely, by selecting appropriate connections from the input neurons and other processing neurons.

References

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[4] The index -1 indicates spikes received from the input layer.

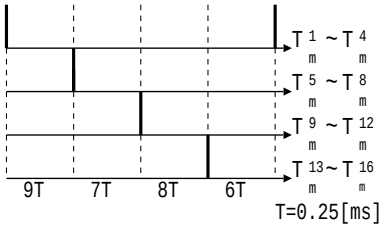

Figure 1: Input data.

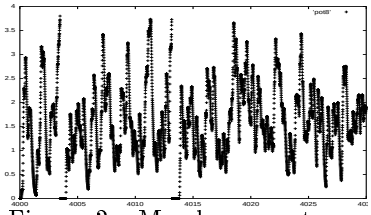

Figure 2: Membrane potential of the 8th neuron (before learning).

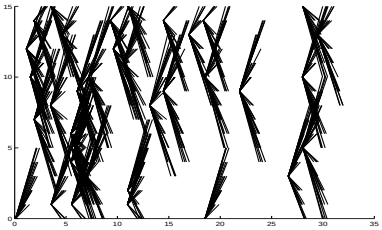

Figure 3: Spike transmission diagram 1.Lines from the firing time to the arrival time (before learning).

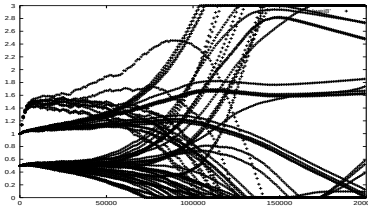

Figure 4: Time-series of the transmission efficacy $J_{8,x}$.

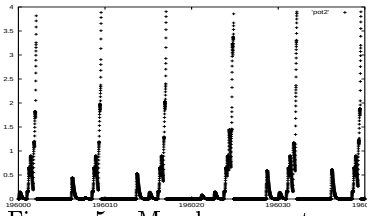

Figure 5: Membrane potential of the 2nd neuron (after learning).

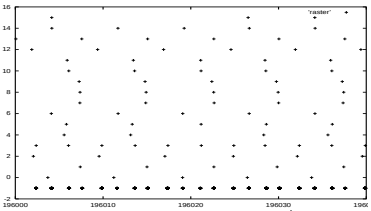

Figure 6: Spike raster (after learning).

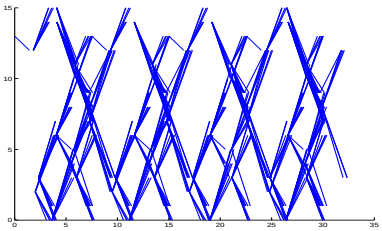

Figure 7: Spike transmission diagram 2(after learning). Weak inputs are ignored.

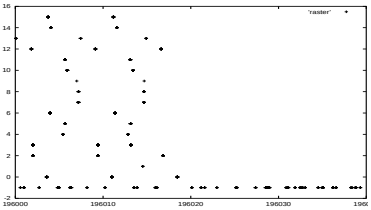

Figure 8: Response disappears for an adverse input sequence.