Weight Distributions of the Third and Fifth Order Reed-Muller Codes of Length 512

Tsukasa Sugita*, Tadao Kasami*, and Toru Fujiwara†

Abstract — The weight distributions of the third and fifth order Reed-Muller codes of length 512 are presented.

Index Terms — Weight distribution, Reed-Muller code, Coset.

1. Introduction

Let $RM(r, m)$ denote the r-th order Reed-Muller code of length 2^m. It is stated in [1] that the smallest Reed-Muller codes for which the weight distributions are not presently known (in 1977) are $RM(3, 9), RM(4, 9)$ and $RM(5, 9)$. This correspondence presents the weight distributions of $RM(3, 9)$ and $RM(5, 9)$, the dual code of $RM(3, 9)$.

2. Background

For linear block codes A and B such that $A \supseteq B$, let A/B denote the set of cosets in A with respect to B, and let $[A/B]$ be the set of chosen coset leaders of A/B. For two binary sequences $\bar{u}$ and $\bar{v}$, let $\bar{u} \circ \bar{v}$ denote the concatenation of $\bar{u}$ and $\bar{v}$. For codes A and B, let $A \circ B$ be defined as $\{\bar{u} \circ \bar{v} : \bar{u} \in A \text{ and } \bar{v} \in B\}$. The following facts (F1) to (F4) are used for computing the weight distribution of $RM(3, 9)$.

(F1) [Example 3 in [2, 3]] For linear block codes C_1 and C_2 such that $C_1 \supseteq C_2$, let C be a linear block code constructed by $|\bar{u}|\bar{u} + \bar{v}|$ construction [1] from C_1 and C_2, that is,

*Graduate School of Information Science, Nara Institute of Science and Technology, Ikoma, Nara 630-01, Japan

†Department of Information and Computer Sciences, Osaka University, Toyonaka, Osaka 560, Japan

$C \triangleq \{\bar{u} \circ (\bar{u} + \bar{v}) : \bar{u} \in C_1 \text{ and } \bar{v} \in C_2\}$. Then $C = \{(\bar{u} + \bar{v}_1) \circ (\bar{u} + \bar{v}_2) : \bar{u} \in [C_1/C_2], \bar{v}_1 \in C_2 \text{ and } \bar{v}_2 \in C_2\}$ and therefore $\{\bar{u} \circ \bar{u} : \bar{u} \in [C_1/C_2]\}$ can be chosen as $[C/(C_2 \circ C_2)]$. Let $\{B_1 = C_2, B_2, \cdots, B_s\}$ be a partition of C_1/C_2 such that, for each block B_i with $1 \leq i \leq s$, every coset in B_i has the same weight enumerator $W_i(x, y)$, and let ν_i denote the number of cosets in B_i. Then, the weight enumerator $W(x, y)$ of C is given by

$$W(x, y) = \sum_{i=1}^{s} \nu_i W_i^2(x, y). \tag{1}$$

(F2) It is known [1] that $RM(r, m)$ is constructed by $|\bar{u}|\bar{u}+\bar{v}|$ construction from $RM(r, m-1)$ and $RM(r-1, m-1)$ with $0 < r < m$ and the general linear group $GL(m, 2)$ permutes the codewords of $RM(r, m)$. Two cosets in $RM(r, m)/RM(r-1, m)$ are said to be $GL(m, 2)$-*equivalent* if and only if one is a permutation of another by a transformation of $GL(m, 2)$. Cosets in the same $GL(m, 2)$-*equivalent* class, orbit, have the same weight distribution. Let $s(r, m)$ denote the number of $GL(m, 2)$-*orbits* in $RM(r, m)/RM(r-1, m)$ for $0 < r \leq m$, and suppose that the $GL(m, 2)$-*orbits* are numbered 1 to $s(r, m)$ and the first orbit consists of $RM(r-1, m)$, only. For $1 \leq i \leq s(r, m)$, let $\nu_i(r, m)$ denote the number of cosets in the i-th $GL(m, 2)$-*orbit* in $RM(r, m)/RM(r-1, m)$ and let $W_{r,m,i}(x, y)$ and $a_{r,m,i}(w)$ denote the weight enumerator of a coset in the i-th orbit in $RM(r, m)/RM(r-1, m)$ and the number of codewords with weight w in a coset in the i-th $GL(m, 2)$-*orbit*, respectively. Then the weight enumerator of $RM(r, m)$, $W_{r,m}(x, y) = \sum_{w=0}^{2^m} A_{r,m}(w) x^{2^m - w} y^w$ where $A_{r,m}(w)$ denotes the number of codewords with weight w in $RM(r, m)$, is given by

$$W_{r,m}(x, y) = \sum_{i=1}^{s(r,m)} \nu_i(r, m) W_{r,m,i}(x, y). \tag{2}$$

It follows from (1) and (2) that for $1 \leq r \leq m-1$,

$$W_{r,m}(x, y) = \sum_{i=1}^{s(r,m-1)} \nu_i(r, m-1) W_{r,m-1,i}^2(x, y). \tag{3}$$

If exactly one vector is chosen in a coset of each $GL(m, 2)$-*orbit* in $RM(r, m)/RM(r-1, m)$, then such a vector is called a representative of the $GL(m, 2)$-*orbit*.

(F3) X.Hou has shown (1) $s(r, m)$ for $6 \leq m \leq 11$ and $1 \leq r < m$, (2) $\nu_i(3, 6)$ and $W_{3,6,i}(x, y)$ with $1 \leq i \leq 6 = s(3, 6)$ and (3) a set of representatives of the $GL(m, 2)$-*orbits* in $RM(3, m)/RM(2, m)$ with $6 \leq m \leq 8$ in [4], and $W_{3,8,i}(x, y)$ with $1 \leq i \leq 32 = s(3, 8)$ in [5].

3. Computation of the Weight Distribution of RM(3,9)

In order to compute the weight distribution of $RM(3,9)$ by using (3), we need to obtain $\nu_i(3,8)$ for $1 \le i \le 32 = s(3,8)$. Since $W_{3,8}(x,y)$ is known [6, 7, 8, 9], we have the following linear equations for $\nu_i(3,8)$ with $1 \le i \le 32$ from Eq.(2):

$$\sum_{i=1}^{32} \nu_i(3,8) a_{3,8,i}(w) = A_{3,8}(w), \quad \texttt{for} \;\; 0 \le w \le 256. \tag{4}$$

The number of linearly independent equations in (4) turns out to be 16. Since formulas for $A_{3,9}(w)$ with $0 \le w \le 159$ and $A_{5,9}(w)$ with $0 \le w \le 39$ are known [9], we have the following additional equations from Eq.(3) and the MacWilliams identities [1]:

$$\sum_{i=1}^{32} \nu_i(3,8) \sum_{j=0}^{w} a_{3,8,i}(j) a_{3,8,i}(w-j) = A_{3,9}(w), \quad \texttt{for} \;\; 0 \le w \le 159, \tag{5}$$

$$2^{-130} \sum_{h=0}^{512} P_w(h) \sum_{i=1}^{32} \nu_i(3,8) \sum_{j=0}^{h} a_{3,8,i}(j) a_{3,8,i}(h-j) = A_{5,9}(w), \quad \texttt{for} \;\; 0 \le w \le 39, \tag{6}$$

where $P_w(h)$ is a Krawtchouk polynomial [1]. Since the number of linearly independent equations in (4) to (6) turned out to be 31, we used the following fact (F4).

(F4) Let P_m be the set of Boolean polynomials of variables $X_1, X_2, \cdots, X_m$. We use the representation of a codeword of a Reed-Muller code by a Boolean polynomial [1]. Then $RM(r,m)$ is the set of polynomials of degree r or less in P_m. For $f \in P_m$, let $m(f)$ denote the minimum of n for which there are a transformation $T \in GL(m,2)$ and a polynomial $g \in P_n$ such that $T(f) \equiv g \pmod{RM(r-1,m)}$. For $1 \le i \le 12 = s(3,7)$, a representative f_i of the i-th $GL(m,2)$-*orbit* of $RM(3,8)/RM(2,8)$ shown in [Appendix in [4]] has $m(f_i)$ less than 8. For f and g in P_m such that $m' \stackrel{\triangle}{=} \max\{m(f), m(g)\} < m$, $f + RM(r-1,m')$ and $g + RM(r-1,m')$ are not $GL(m',2)$-*equivalent* if and only if $f + RM(r-1,m)$ and $g + RM(r-1,m)$ are not $GL(m,2)$-*equivalent*. By using this property, we obtained the exact value of $m(f_i)$ with $1 \le i \le 12$. For $f \in RM(r,m)$, let $\nu(r,m,f)$ denote the number of cosets in the orbit containing $f + RM(r-1,m)$ in $RM(r,m)/RM(r-1,m)$. Then it is proved in Appendix that

$$\nu(r,m,f) = \nu(r,m(f),f) \prod_{i=0}^{m(f)} (2^{m-i}-1)/(2^{m(f)-i}-1). \tag{7}$$

For $1 \le i \le 12$, let the $GL(7,2)$-*orbit* in $RM(3,7)/RM(2,7)$ which contains $f_i + RM(2,7)$ be numbered i. Since $W_{3,7}(x,y)$ and $W_{3,7,i}(x,y)$ are shown in [10] and [11] respectively, we first

computed $\nu_i(3,7)$ with $1 \leq i \leq 12$ by the following equations:

$$\sum_{i=1}^{12} \nu_i(3,7) a_{3,7,i}(w) = A_{3,7}(w), \quad \texttt{for} \;\; 0 \leq w \leq 64, \tag{8}$$

and some equations of the following type:

$$\sum_{i=1}^{12} \nu_i(3,7) \sum_{j=0}^{w} a_{3,7,i}(j) a_{3,7,i}(w-j) = A_{3,8}(w). \tag{9}$$

We next computed $\nu_i(3,8)$ with $1 \leq i \leq 12$ by the value of $m(f_i)$ and Eq.(7). We solved the remaining unknowns $\nu_i(3,8)$ with $13 \leq i \leq 32$ by using (4), (5) and (6). The values of $\nu_i(3,8)$ with $1 \leq i \leq 32$ are shown in Table 1. Since $A_{r,m}(w) = A_{r,m}(2^m - w)$ for $0 \leq w \leq 2^{m-1}$, nonzero $A_{3,9}(w)$ and $A_{5,9}(w)$ with $0 \leq w \leq 2^8$ computed by Eq.(3) and the MacWilliams identities are tabulated in Tables 2, 3 and 4, respectively. The computational results were obtained by using *Mathematica*[1] on an *ONYX*[2] system at Nara Institute of Science and Technology.

4. On the Weight Distribution of RM(4,9)

Since $s(4,8) = 999$ [4] and no set of representatives of cosets in $GL(8,2)$-*orbits* in $RM(4,8)/RM(3,8)$ has been found, it is not easy to apply the method stated in sections 2 and 3 to computing the weight distribution of $RM(4,9)$. Since $RM(4,9)$ is a doubly-even self-dual code [1], the theorem due to Gleason [1] can be applied. Weight enumerator $W_{4,9}(x,y)$ can be expressed as

$$W_{4,9}(x,y) = \sum_{i=0}^{21} g_i [G_1(x,y)]^{64-3i} [G_2(x,y)]^i,$$

where $g_i \in \{0,1\}$, $G_1(x,y) = x^8 + 14x^4y^4 + y^8$ and $G_2(x,y) = x^4y^4(x^4 - y^4)^4$. By using the values of $A_{4,9}(4j)$ with $0 \leq j \leq 19$ computed by the formulas in [9], coefficients g_i with $0 \leq i \leq 19$ is tabulated in Table 5 and if two more $A_{4,9}(4j)$, e.g. $A_{4,9}(80)$ and $A_{4,9}(84)$, are found, then $W_{4,9}(x,y)$ can be determined.

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[1] *Mathematica* is a trademark of Wolfram Research, Inc.

[2] *ONYX* is a trademark of Silicon Graphics Inc.

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Appendix A

Proof of Formula (7)

Suppose that $f \in RM(r,m)$ and $m(f) < m$. Without loss of generality, we assume that only variables $X_1, X_2, \cdots, X_{m'}$ occur in f, where $m' = m(f)$. For $m' \leq h \leq m$, let I_h be defined as

$$I_h \stackrel{\triangle}{=} \{T \in GL(h,2) : T(f) \equiv f \pmod{RM(r-1,h)}\}. \tag{A.1}$$

Let $t_{i,j}$ with $1 \leq i \leq m$ and $1 \leq j \leq m$ be the (i,j)-entry of the coefficient matrix of a transformation $T \in GL(m,2)$. Let $T_{m'}$ denote the linear transformation whose coefficient matrix is $(t_{i,j} : 1 \leq i \leq m', 1 \leq j \leq m')$.

We first prove that $T \in I_m$ if and only if

$$(1) \qquad T_{m'} \in I_{m'}, \tag{A.2}$$

$$(2) \qquad t_{i,j} = 0, \qquad \texttt{for } 1 \leq i \leq m' \texttt{ and } m' < j \leq m. \tag{A.3}$$

The if part is obvious. Suppose that $T \in I_m$. Let $f' \in RM(r,m)$ be defined as

$$f'(X_1, X_2, \cdots, X_m) = f(\sum_{j=1}^{m} t_{1,j}X_j, \sum_{j=1}^{m} t_{2,j}X_j, \cdots, \sum_{j=1}^{m} t_{m',j}X_j). \tag{A.4}$$

Then,

$$f'(X_1, X_2, \cdots, X_m) \equiv f(X_1, X_2, \cdots, X_{m'}) \pmod{RM(r-1,m)}. \tag{A.5}$$

Since $f'(X_1, X_2, \cdots, X_{m'}, 0, \cdots, 0) \equiv f(X_1, X_2, \cdots, X_{m'}) \pmod{RM(r-1,m')}$, it follows from the definitions of $m(f)$, $T_{m'}$ and f' that $T_{m'} \in I_{m'}$. For a binary m'-tuple $\bar{b} = (b_1, b_2, \cdots, b_{m'})$, let $\Delta_{\bar{b}} f(X_1, X_2, \cdots, X_{m'}) \stackrel{\triangle}{=} f(X_1+b_1, X_2+b_2, \cdots, X_{m'}+b_{m'}) + f(X_1, X_2, \cdots, X_{m'}) \in RM(r-1,m')$. It follows from (A.5) that

$$f'(X_1, X_2, \cdots, X_{m'}, \underbrace{0, \cdots, 0}_{j-m'-1}, X_j, \underbrace{0, \cdots, 0}_{m-j}) \equiv f(X_1, X_2, \cdots, X_{m'}) \pmod{RM(r-1,m)}. \tag{A.6}$$

Define $X' = \sum_{l=1}^{m'} t_{i,l}X_l$ with $1 \leq i \leq m'$ and $\bar{t}_j = (t_{1,j}, t_{2,j}, \cdots, t_{m',j})$. Since $T_{m'} \in I_{m'}$, it follows from the definition of $\Delta_{\bar{b}} f$ that

$$\begin{aligned} & f'(X_1, X_2, \cdots, X_{m'}, \underbrace{0, \cdots, 0}_{j-m'-1}, X_j, \underbrace{0, \cdots, 0}_{m-j}) \\ &= f(X'_1 + t_{1,j}X_j, X'_2 + t_{2,j}X_j, \cdots, X'_{m'} + t_{m',j}X_j) \end{aligned}$$

$$\equiv f(X'_1, X'_2, \cdots, X'_{m'}) + \Delta_{\bar{t}_j} f(X'_1, X'_2, \cdots, X'_{m'}) X_j \pmod{RM(r-1, m)}$$

$$\equiv f(X_1, X_2, \cdots, X_{m'}) + \Delta_{\bar{t}_j} f(X'_1, X'_2, \cdots, X'_{m'}) X_j \pmod{RM(r-1, m)}. \quad \text{(A.7)}$$

From (A.6) and (A.7), $T \in I_m$ only if for $m' \leq j \leq m$,

$$\Delta_{\bar{t}_j} f(X'_1, X'_2, \cdots, X'_{m'}) \equiv 0 \pmod{RM(r-1, m)}. \quad \text{(A.8)}$$

From the definition of $\Delta_{\bar{t}_j}$, the condition (A.8) can be rewritten as $\Delta_{\bar{t}_j} f(X_1, X_2, \cdots, X_{m'}) \equiv 0 \pmod{RM(r-1, m')}$. Since $m(f) = m'$, it follows from Lemma 5.2 in [5] that $\bar{t}_j = 0$ for $m' < j \leq m$. From (A.2) and (A.3), we have that

$$|I_m| = |I_{m'}| \prod_{i=m'}^{m-1} (2^m - 2^i). \quad \text{(A.9)}$$

Since $|GL(j,2)| = \prod_{i=0}^{j-1}(2^j - 2^i)$ [1], $\nu(r, m, f) = |GL(m,2)|/|I_m|$ and $\nu(r, m(f), f) = |GL(m',2)|/|I_{m'}|$,

$$\nu(r, m, f) = \nu(r, m(f), f) \prod_{i=0}^{m'-1} (2^m - 2^i) / \prod_{i=0}^{m'-1} (2^{m'} - 2^i). \quad \text{(A.10)}$$

Table 1: The Values of $\nu_i(3,8)$.

i	$\nu_i(3,8)$	i	$\nu_i(3,8)$	i	$\nu_i(3,8)$	i	$\nu_i(3,8)$
1	1	9	60717988800	17	116060411658240	25	3626887864320
2	97155	10	4533609830400	18	1740906174873600	26	55266862694400
3	84330540	11	113340245760	19	6963624699494400	27	497401764249600
4	3855110400	12	3454178918400	20	12379777243545600	28	108806635929600
5	5059832400	13	2763343134720	21	27854498797977600	29	18134439321600
6	1799051520	14	64478006476800	22	15916856455987200	30	2321208233164800
7	449762880	15	290151029145600	23	1740906174873600	31	1740906174873600
8	566701228800	16	2266804915200	24	13600829491200	32	217613271859200

Table 2: Weight Distribution of $RM(3,9)$.

w	The number of codewords of weight w, $A_{3,9}(w)$
0	1
64	6304280
96	91931532672
112	27817105940480
120	29533455515648
128	12583593752692060
136	361519028967047168
144	17833197403694489600
148	12590654537544499200
152	1020786087075313090560
156	4318594506377763225600
160	76250672761991902699392
164	756987922760787925401600
168	106817818361651107410739 20
172	14116041796899633132208 1280
176	18812934593637648668214886 40
180	22693173473127523177526722560
184	24342579651884676892807741440
188	22869599995308270433430038118 40
192	18794346413747525588771309881256
196	13516644986672653098444725944 3200
200	85178270474362350560724434026 4960
204	47084585094754567843220193017856 00
208	22852391704572296323834439067729920
212	97462433014907637029830197227028480
216	36552627818416194695840339638144 2048
220	12063231789211397866681432712360 75520
224	35054016131639341517114623232067673 60
228	89736775872820393628228931931235942 40
232	20247351711512218010880337711776923648
236	40281223524945861679013088343023943680
240	70684468756337914215018072966269648896
244	10943314701807506256151504766361403392 0
248	14951109074794192233081664718029979648 0
252	18028537409432319287236313019070808064 0
256	19189023549869311346107486308760925024 6

Table 3: Weight Distribution of $RM(5,9)$ Part 1:$A_{5,9}(w)$ for $0 \leq w \leq 138$.

w	The number of codewords of weight w, $A_{5,9}(w)$
0	1
16	105911904
24	1838630653440
28	509913567887360
30	1721378550054912
32	131916883632582384
34	2108688723817267200
36	74239828158397841408
38	1712014250742613278720
40	63207760722959840830976
42	2147443289652067471196160
44	90627109352815685973442560
46	43522004773831765036388843 52
48	24902390354863481392150114 5120
50	16050701372137033277449458155520
52	10968871629040406366425348339138 56
54	74810717240262914957714979500851200
56	49031650270220359891008994479440865 28
58	30278276184839439770782971336171978 7520
60	17467153467742338336685042904470170697728
62	93865867870066054024603779624850985425 3056
64	46978315038317695971196368807565028283143800
66	21917948352689502498994549241765615657969254 40
68	95459675749200894268563971091367024118799073280
70	38870804310376060399786672992234295915098648084 48
72	14821072616815048697056683867718880321069379278 5408
74	52995257642958089581626229585798032689375926969958 40
76	17795728571565643712210058047539001141941181885539942 4
78	56196012907436606062764646680346143022337911974946406 40
80	16709590745414531551326939557066156254876020195784660 2208
82	46841185646451920826931695425042432611201002253039802777 6
84	12393564269145800704388922691428465616707359436178404868096 0
86	30984927369911418790700805122893375958621675263934479472087040
88	73273524959675858950275434414894525325131323470601564537090560
90	16406683183259008568747191916434548711247583993516799864235622 40
92	34816580824880857988134288998287533775431566447944390688176439296
94	70087187063261763040397750576136623227619333292001408352362954752 0
96	13395413627264244819475448825978140267478338865248169990147726119632
98	24327626852581995872387163594442819567790296510373233711713282424832 0
100	42016023176852057460454466057363920799715369094186147781583944297676 80
102	69060932213003315454984745583010974767136826436524028274829424238526464
104	10811078904778806002678131082112195950232617353897752001622606934050432 00
106	16129780041437260359771534708753055515294128680680239439958453806065254400
108	22951018797278735891439867670740504002219203425170815870889140216382750720 0
110	31165148227043737231782453358627330991202730487953817619418212809195535728 64
112	40410909141698065708805110881008816383790802838796352896251645997071431460256
114	50066613095909105739365658036212452369705679879021211006134262481631991300096 0
116	59301538746868172092763757630504415913290353729869227042098641366409344505610 24
118	67187792921810223893827412985390037813455973786057624346242005308174262106849280
120	72853587056014976901529572808387776021524051645705262543613619276040616186016768 0
122	75642968400281313787255267937502345088672322604286210023910247849151470847498649 60
124	75241245318690520022704645887548108129890438249628832023995987412379674474059530240
126	71732853536973293486304962026970039030833935533687582853725696452660507195816594636 8
128	65577136836427172397480003125603740809727890805323936308132350197773039935984991608 60
130	57510797071001890869778530700170351404932683090549148998588550741503850105154629009408
132	48405253454243333339005399591520686915807849106597145167214017931549186160048845212876 8
134	39116398846819239521294495649936692664334361143773472539692450737447259800076288000000 0
136	30361228399045874440201922320911094153985466933290295218470354589487409553565713002530816
138	22643251900272232603767164083228165816714627970733228339166058722038324742676423263649792 0

Table 4: Weight Distribution of $RM(5,9)$ Part 2:$A_{5,9}(w)$ for $140 \leq w \leq 256$.

w	The number of codewords of weight w, $A_{5,9}(w)$
140	162321630349012178452759799886380923576848973091093917793273307518734780859712662643489216 0
142	111888586793166860316763645815479443248528409968707352348343631217454218876527001610863247360
144	74184871575714216390092189650930090312447286229338992432728015836058942525457751564530925280
146	47326934414405570187908790380999564392805094573844094544008845685173006580268981081596965683 2
148	29060512816788196917644477783274065145016227572234826790301330271418057847230418574702503526 40
150	17180419147686165705244741030612218501021159601525095309154176504428701242861092608223154798592
152	97820300412074046126385208955413774509337005271260432645251746426578469079643929807052551704064
154	53655443617929079540675769530482168940474847601195202115548010000161371945621232925225460432896 0
156	28360163883511347972603835523320500873349482123872598258803644761217516517264998322936894802165 76
158	14448752364743062794395197597982309330668799935298191332628269351694021638073418833127345917788160
160	70972680542571643589355844269752740268617435840477523221919027293596475858883641990208271872236 4912
162	33620184902982178164067530381364269409926127560524318659673190792451077632154142139568867730522112 0
164	15362507803004911950998237462098465918025294507971485317353084840936524645351231645252208199448985 60
166	67729645573554623861071309351410695049070440023868359633879228642756619976466403088132188509191536 64
168	28816965326900539814286007258584854224642090581978215003074977541721251434255365320873924447328273920
170	11834916021063865275904053492139188846263985026092501723898005737432330952516796378048919288230497484 8
172	46926818176543930919573049312365050214167978916786693784654291792675543008960974672009226936163250995 2
174	17968191691676478233306722688937546412050414321758071465542870656172646130472274860742380913134508441 60
176	66450806585457822391007647876902575229206049272825135314091476898182430776000702149873392680533308957 76
178	23740566207259355323848856868522484491300314541809446106845277125707980945759521477678698203100594831 360
180	81951373516567350025733009268384901773522548277749784235179332003419211707037966882891517706089938944000
182	27338353282990162191208341492707659366078670827882101491361645888664393337220114189769093009069588034355 2
184	88148165120403953109393253619178169965333560121467246055776312551336502438988597085862477166860327499264 0
186	27475790753135851190645401947625392484157783602636907730228391917471832510537924376817449653720567848632 32
188	82804074135133218615567195822912971737574715339970170424699174547739605834902508194470820248278425308364 80
190	24131473033667395139393868496962558757218686040462651584277758384406850625967392441824402600133280850247680
192	68015851758996763099858994207656979670452799481029516599795142173267898007661504662133890969377741582313800
194	18543502343780753104090609819768697905278252918254813422929288379298975910653165150355318227694349961338880
196	48908851315205719450836159432013947689870122877773628724050741796843855886200008283972715593476397715885376
198	12481123570516272660965572757684981458312815630329402662075752974151493396508054763899732152669257100322406 40
200	30820848913504530393593427682683395245454194923351433999699374364974959562653750861774154120807517340566476 80
202	73656682226864972000176283801442129821337692803932167281506958010626393104946257551322304476738685443055288 32
204	17037509877599236134204786596494114823995150822924355238198255706516200313271102623622832888646483186295111680
206	38148207056269793320053701193892810551776903340513206063226303011173836696973923190400252830345231535123202048
208	82691661198477791958393996944119601792890327297752669002314340798752883897207427882079897223274755376859503200
210	17354509674901313221398012865155508064412185592376518581284308329694832332788366825756947617360650766495042764 8
212	35266914925956343880254005308691023882302747564796680983379258334259745987663614424561583895694708779227296563 2
214	69401129148749156378807079026580309912371429198231153469233034329498568942555740609402313378098476840938110976 0
216	13226564031953472942426139828437800695856887571045312281751097180760007249270993441852918884910215908084767216 64
218	24414314701521524908735689549300065068846797946849274453512701834339302563265179418564478014124197519036933734 40
220	43650848838075284323128056644993911886033997087318031165134460561822012162360805330542839009688787081198822686 72
222	75600259416023257500811977278513363803537540909931402969202945012589123287416691467882645460345305708865079214 08
224	12684292404021679234351080668866520254031122994537759592829734019566857977810590472282778634185026922365981337680
226	20618148927174354351908022099623029911120282817895810463756188841722735668055852606581106694390406425194012868608
228	32471313839052121941881576654692659631279471468626115235861851245411609573159905606450177792241299440620702105600
230	49549732976500800165424455646306408334046219201185100466057093272175701680357870871112370551485606385673924771840
232	73265038448348193885441198580471194409170578229067729415016925722832883899375418763135654876150004966808840296960
234	104975327456530591437046814737287875070659117036351852338295538175979801335974889970843100576281623997232353116160
236	145757844682971415870920068800208984401649704386589465822826425933295059675361951733282048281545691882476382191616
238	196131979070267887540382817819662132091969755984125756909146554617875949870376170122627614448525602225275328424960
240	255771692789647463803969936167126336753867997153535937644963143364675693964663645739538314309439143571009205603616
242	323264686034609475873910907286293612955678874526033650833566109722707458282433163165348720202037228959501658357760
244	395984519778278456330063907377108296931931208486062463638262995008788408174731839503935694230619158517285154979840
246	470135528409731097082363579828710168282418639365780648560813652899223408680122793527261829454139226894394470694912
248	541005834491346889012273226167653452406027126642147352925777775889331335561255485180981509115319612804551887155200
250	603423567877963007195183207056586568818503967951309908919870353210803539232404953835229489169502003061042406490112
252	652363726342738037659220547412627307525881454423004992683328775148541430937870800292417684426187333824133185929216
254	683610428121128807942048359260003166023558368413146017229453875599442034925053937100585736501155774269230950645760
256	694354672901341402258049303141755061046426499149485433402231129386898787297233282882711912960407961573509319588422

Table 5: The Values of g_i.

i	g_i	i	g_i
0	1	10	560488125281758654464
1	-896	11	-11885355149550800420864
2	366400	12	184790978198497210204160
3	-90591488	13	-20575111083511137030602752
4	15125312416	14	15886111690015316194099200
5	-1804090983680	15	-81428598375642719259197440
6	158633285104000	16	26105395318391905423818752O
7	-10462864561216000	17	-48139848578651260565061632O
8	521844917396785920	18	45135226633071414939117158 4
9	-19703740374727094272	19	-17672713137775132319888179 2