Evolution of Associative Memory with Environmental Change

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Abstract

There have been a lot of researches which apply evolutionary techniques to layered neural networks. However, their applications to Hopfield neural networks remain few so far. We have been applying a Genetic Algorithm to fully connected associative memory model of Hopfield, and reported elsewhere that the network can store some number of patterns only by evolving weight matrices with the Genetic Algorithm. In this paper we present that evolving not only weight matrices but also patterns to be stored results in the higher capacity of associative memory than the case of fixed patterns above. From the Artificial Life point of view, the patterns to be stored to the network is considered to be an environment. And we could derive interesting phenomena from evolving both environment and individual's ability.

1 Introduction

Associative memories are the process in which a complete or a noisy input of the predetermined stored patterns results in the retrieval of them. They have the error-correcting capability because of their distributed storage of patterns among neurons, though they have a limited capacity. Those networks are one possibility to be used as a memory organ of digital organisms of

our Artificial Life research in the future. However, as a first step, we are now investigating basic behavior of the function of associative memory under the evolutionary process. We are now using a simple Genetic Algorithm ([1], [2]) to evolve the networks. Genetic Algorithm has been applied to layered neural networks a lot (see for example [3]), but curiously has not been attempted for Hopfield model. Probably this is because the Hopfield network has a fixed architecture. However, as we reported previously [4], Genetic Algorithm can enlarge the capacity of *Hebb-rule associative memory* by pruning some of its connections. Hence the Genetic Algorithm may be used also as a method to determine the architectures of Hopfield networks as is effective in layered networks.

We also reported [5] that it can make a network store some number of patterns without using any learning algorithms.

In the context of Artificial Life, it has been shown that environmental change enhances the performances [6], [7]. Here we give the patterns to be stored an opportunity to be modified so that the fitness of individuals are improved in every certain generations. When we take the patterns as a language of individuals in a population, it is interesting to see if slight occasional modifications of their language enhance the individual's adaptability.

The goal of this research is twofold. One is to improve the techniques to enlarge the associative memory capacity which are proposed so far, and the other is to know if the ability of associative memory can be adaptive to the changing environment.

In the following section, we provide an outline of our experiment, a brief review of associative memory and the implementation of our Genetic Algorithm to Hopfield model of associative memory. The difference from our previous implementation is described in detail.

2 Experiment

We apply a simple Genetic Algorithm to fully connected associative memory model of Hopfield. To be more specific, an auto-associative discrete-time asynchronous-update bipolar-state network. The connection weights are chosen randomly either from -1 or 1 at the start of the Genetic Algorithm, therefore the network can not store any patterns initially. The objective of the experiment is to make this network store some number of patterns without any learning algorithm besides the Genetic Algorithm.

We set the size of the network to 49 neurons in this paper, considering computational expense. This size of *Hebbian-rule associative memory* would have a capacity of seven patterns at most. In our previous simulation [5], we succeeded in storing eight patterns to the network of same size only by means

of Genetic Algorithm. In this new version of the experiment, we occasionally change the patterns to be stored to see if it enhances the capacity.

From Artificial Life point of view, each individual in the population tries to memorize a given set of patterns, increasing its accuracy, generation by generation. And our concern in this paper is how these occasional changes in language affect the accuracy and the capacity.

Before describing the implementation of our Genetic Algorithm, let us review some fundamental concepts relating to associative memory.

Associative Memory

Hopfield network which has N neurons can store some number of N-bit bipolar patterns, if all the connection weights are determined adequately. The patterns are n-dimensional random vectors whose elements take the values ± 1 randomly with equal probability. The μ-th pattern is denoted by

$$\xi^\mu = (\xi_1^\mu, \xi_2^\mu, \cdots, \xi_i^\mu, \cdots, \xi_N^\mu),$$

where, ξ_i^μ is the i-th bit of the μ-th pattern which takes the value of either -1 or 1. To store these p patterns, each connection weight has to be adequately adjusted. If we use Hebbian rule, for example, the weight w_{ij} is determined as follows:

$$w_{ij} = \sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu.$$

This process of determining w_{ij} is referred to as *learning*.

These w_{ij} define a Liapnov (energy) function on an N-dimensional bipolar state space, i.e.,

$$E(S) = \frac{1}{2} \sum_{i,j(i \neq j)} w_{i,j} S_i S_j,$$

where S moves over all possible combinations of N dimensional vector $\{-1, 1\}^N$ and S_i is the i-th component of the state S. If the number of stored patterns is small enough, this function takes a local minimum where S corresponds to one of stored patterns. In this sense the stored patterns are said to be *attractors*, and the size of this local minimum is referred to as *basin of attractor*. As the number of stored patterns p increases, spurious minima become non-negligible and finally these local minima are randomized.

The retrieval of one of the stored memories is as follows. The state of each neuron is asynchronously updated by

$$s_i^\mu(t+1) = sgn(\sum_{j=1}^{N} w_{ij} s_j^\mu(t)),$$

where $s_i^\mu(t)$ is the state of the i-th neuron at time t when the μ-th pattern is given to the network. For each neuron, the weighted sum of its inputs is computed, and if this is greater than or equal to zero, the state of the neuron takes the value of 1, and -1 otherwise. At each step of updating, a neuron is chosen according to pre-assigned order (once in a cycle), instead of chosen randomly. For the remainder of this paper, the term "update" will refer to this asynchronous update.

When input is one of stored patterns which is given small noise within the size of basin of attractor, this input pattern relaxes to the original stored pattern after several steps of the update. In this simulation, however, we do not give any noise to input patterns. In this case, when one of a stored patterns is given to the network, all the states remain unchanged from the start, and the patterns are said to be memorized as *fixed points*. We evaluate the ability of weight matrix by the upper bound of the number of patterns stored as fixed points.

When Hopfield [8] proposed a neural network model as an associative memory system, in which he used Hebbian learning rule, he estimated the memory capacity to be at most 15% of the number of neurons with using computer simulations. Later the capacity was verified more analytically by Amit et al. [10] with spin-glass theory, and by Amari et al. [11] with statistical neurodynamical method. Since Kohonen [12] proposed several types of connection matrices for associative memory to improve the capacity, other various successful methods have been proposed for that. For example, Yanai et al. [13] made it by introducing sparse connectivity, Morita [14] by replacing the sigmoid function with a non-monotonic transfer function, and Willshow et al. [15], Palm [16] and Meunier et al. [17] by using sparsely coded patterns instead of random patterns (the last issue is returned to later). Those mechanisms, however, are not well understood yet. Here we are exploring them by means of Genetic Algorithms.

To know more about associative memory, see for example [18].

Genetic Algorithm

In this subsection we describe the implementation of our Genetic Algorithm briefly. As in Figure 1, our Genetic Algorithm proceeds as follows:

(1) At first a weight matrix W_0(49x49) is produced randomly so that each component of the matrix w_{ij} are chosen from $\{-1, 1\}$. Hence the matrix does not store any patterns at this moment. This original weight matrix remains unchanged during evolution. **(2)** Then 256 chromosomes are produced also randomly. The chromosome has a fixed length of 2401(=49x49) alleles, and

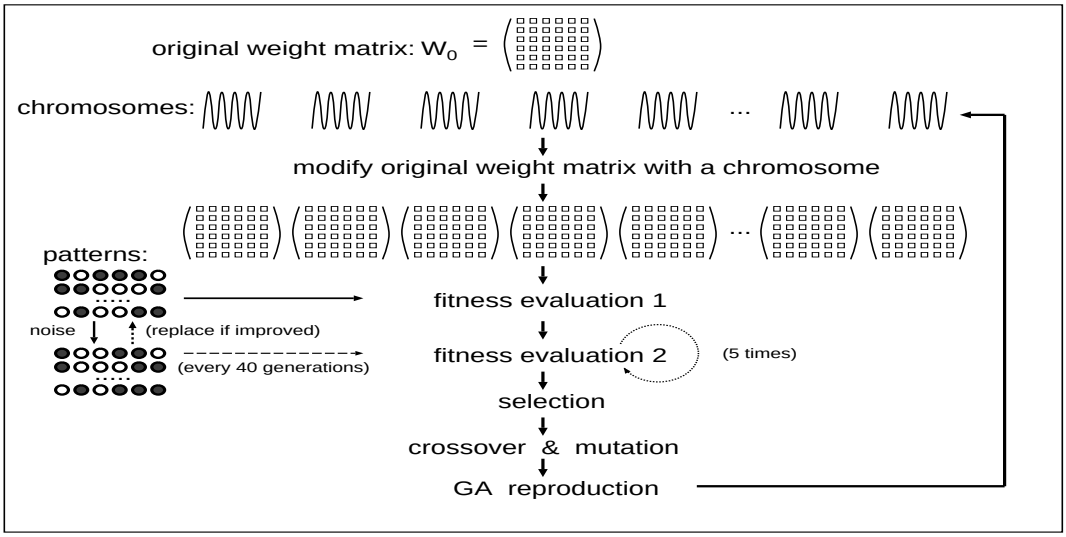

Figure 1: **Schematic representation of our Genetic Algorithm**

its allele values are chosen randomly from $\{-1, 0, 1\}$, where the probability of choosing either -1 or 0 is set to 0.02 here. Each component of the original weight matrix w_{ij} is multiplied by one of these allele values. Allele value 0 implies to prune the connection which is multiplied, and -1 to reverse excitatory/inhibitory connection. We denote the i-th allele value of the n-th chromosome as $c^{(n)}(i)$. **(3)** Each chromosome modifies the original weight matrix W_0, which produces 256 weight matrices slightly different from W_0. The modification was made as follows:

$$w_{ij}^{(n)} = w_{ij} \cdot c^{(n)}(49i + j) \quad (i, j = 0, 1, 2, \cdots, 48; \quad n = 1, 2, 3, \cdots 256),$$

where $w_{ij}^{(n)}$ denotes i-j component of n-th weight matrix in the population. We must note that the larger the population number, the higher the performances. In these experiments, however, we set it to 256 just because of our computer resources. **(4)** Evaluation of fitness value f for each individual weight matrix is made with a set of randomly predetermined patterns ξ^μ (see preceding subsection). First we calculate the total of all the inner products (overlap) of initial states and states at each time of update not more than certain time t_0. And then these are summed up over all sets of initial patterns, i.e.,

$$f = \frac{1}{p \cdot (t_0 - 1) \cdot 49} \sum_{\mu=1}^{p} \sum_{t=2}^{t_0} \sum_{j=1}^{49} \xi_j^\mu s_j^\mu(t).$$

In this paper, t_0 is set to 98 i.e., two times the number of neurons, and we observed that was enough. We must note that fitness 1.000 implies that all

the initial patterns are stored as fixed points, while fitness less than 1.000 includes many possible cases. If we evaluated the fitness at one point of updating time instead, some solutions might be limit cycle. We adopted the fitness evaluation above in order to avoid these oscillated solutions. **(5)** Every 40 generations, we give a perturbation to one randomly chosen bit of each pattern, and for the new set of modified patterns we evaluate fitness of each individual of the generation in the same way as above. If the highest fitness value exceeds the previous value, we replace the new patterns with old ones. This process is repeated five times for the same population. **(6)** Two parent matrices are chosen uniformly at random from the upper 40% of the population which is ranked by fitness. We refer to this upper 40% of the population as upper subpopulation and the rest 60% as lower subpopulation. **(7)** Recombinations are made with uniform crossover. We tested several types of crossover including one and two-point-crossover, and observed that uniform crossover outperformed the others. Furthermore, mutation rotates the value of randomly chosen bit in chromosome $c^{(n)}(i)$ cyclically i.e.,

$$(1) \rightarrow (-1), \quad (-1) \rightarrow (0), \quad (0) \rightarrow (1).$$

The mutation rate is set to 0.01 in this paper. **(8)** Unless highest fitness value reaches the value of one nor generation exceeds 12000, the processes from (2) to (8) are repeated. Then individuals in upper subpopulation (40%) survive to constitute the next generation with their offsprings(60%).

These operations and parameter values were determined mainly on the basis of trial and error.

3 Results and Discussions

In our previous simulation [5], where the patterns to be stored remained fixed during evolution, the network were able to store up to 8 patterns as fixed points. In those simulations, we repeated 30 runs for nine patterns with different random number seed, nevertheless we did not obtain the weight matrix which stored all these nine patterns. So we concluded the associative memory capacity obtained with that method was eight patterns for 49 neurons. On the other hand, new version of the simulation in this paper, where the patterns to be stored were also evolved occasionally, we obtained the weight matrix which stored all the nine patterns as fixed points. Again we repeated the simulation for ten patterns with different random number seed, however none of the best-fitted matrix did not store them as fixed points. We show the fitness vs generation for nine and ten patterns in Figure 2.

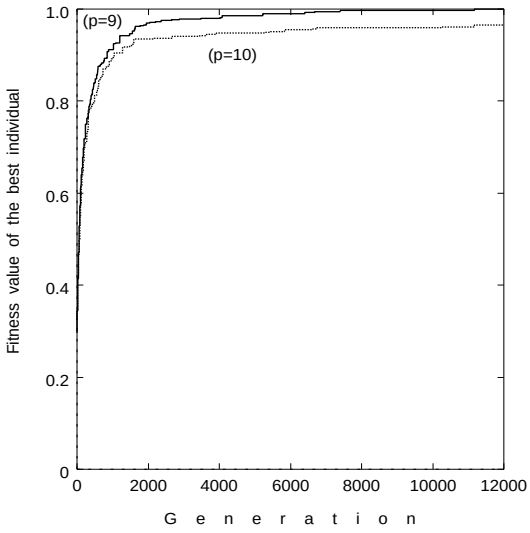

Figure 2: **Fitness vs Generation**

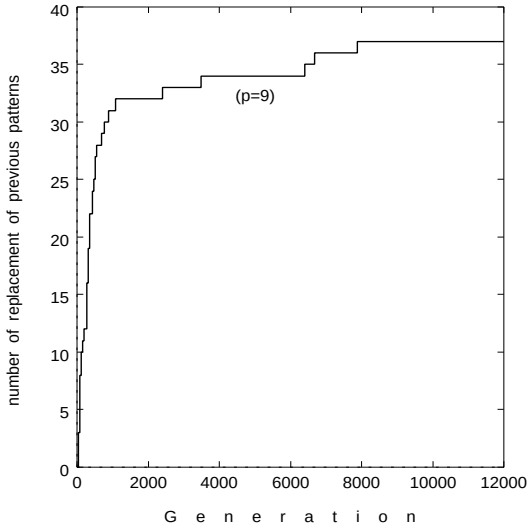

Figure 3: **Number of Replacement of Patterns**

In this new version of the simulation, one bit of each pattern is flipped at every 40 generations, and these modified patterns are checked if they give a higher fitness value to the best-so-far individual. When this occurs, the old patterns are replaced with the new ones.

At early stages of evolution, the modified patterns tend to give individuals higher fitness value. However, as evolution proceeds the number of individual's accepting new patterns decreases asymptotically. In Figure 3 we show the number as evolution proceeds.

It shows that the effect of learning by the GA itself in each generation is very weak. Therefore even the individual which learned those patterns most efficiently have a room to accept the slight changes in patterns. As evolution proceeds, however, accumulated effects of the learning make them difficult to accept the changes.

To ascertain this, we experimented with Hebbian matrix instead of random one as the original weight matrix W_0. However, the modified patterns were never accepted from the first generation at all (not shown in the figures in this paper), probably because Hebbian learning effect is much stronger than the one by Genetic Algorithm.

Some experiments in Artificial Life literatures show that changing environment in every generation enhances the evolvability of individuals (see for example [6] and [7]). In this paper a set of patterns to be given to the individual networks could be taken as an environment. We tried to change the environment in every generation. Although it works at the beginning (until around 100-th generation), the effect is saturated afterwards without attaining fitness of one (not shown here neither).

To see what happens in these replaced patterns, we counted the number of 1's in patterns which were successfully modified and adopted as new patterns. In Figure 4 we show an example of the density of 1's in the replaced patterns as a function of generation numbers. We can see in the figure that these accepted patterns tend to have more 1's than in the previous patterns. So we conjectured that the enhancement in memory capacity is due to diminishing the number of -1's in the patterns.

As we mentioned in the previous section, Willshow et al. [15], Palm [16] and Meunier et al. [17] showed that sparsely coded patterns enhanced the capacity of *Hebb-rule associative memory*. Hence we expected that the density of 1's would be decreased through evolution, though we do not use Hebbian learning. However, we obtained the opposite results here. Yet we do not know the reason for that.

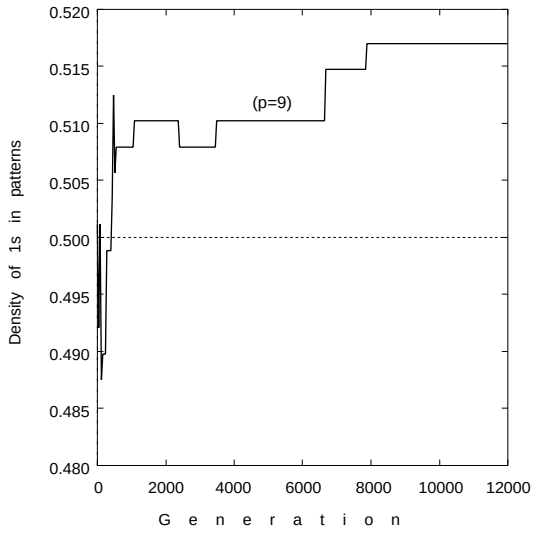

Figure 4: **Density of "1" in Patterns**

4 Conclusions

By slightly modifying patterns occasionally, we obtain a little higher memory capacity than the case of fixed patterns. Acceptance of the modified patterns occur more frequently at early stages of evolution, and asymptotically decreases its chance as generation proceeds. The improvement in capacity is due to the increase of the number of 1's in patterns.

The enhancement in capacity in this method is not so drastic than we expected, comparing to other experiments of changing environment [6], [7]. However, from the Artificial Life point of view, these phenomena observed here might be interesting when we take the patterns as a language of the individuals. Changes in language could be more easily allowed when the culture is young and a little higher rate of "1" is more acceptable in this society. To consider evolvability or elaboration of language, we have to incorporate communication between individuals or species probably.

The result in this paper is just a beginning of our research, we can not draw further conjectures yet. So we will keep researching to obtain further interesting phenomena in Artificial Life context.

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