The B-mode Ultrasound Image Restoration To Degraded Images

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1 Introduction

In a conventional B-mode ultrasonic scanner system, the observation of blood flow by newly developed Digital Subtraction Echography(DSE) technique results in blurred images. The image represents a moving average of the fast moving particles. One method to solve this problem is to use a scanner system with the high speed frame rate. Another is to use image restoration method for the degraded image. The image restoration can also be used to eliminate the noise, especially white noise, in the picture. The methods of inverse and Wiener filtering are described in this paper. The results of restoration to the degraded ultrasound images are also presented.

2 Method

The fundamental model of image restoration can be expressed,

$$g(x,y) = \int\int h(x-x', y-y') f(x', y') dx' dy' + n(x,y) \tag{1}$$

where g is the degraded image, h is the point spread function of the image formation system, f is the ideal image, and n is a two-dimensional random noise process. In the absence of noise, Eq.(1) becomes

$$g(x,y) = \int\int h(x-x', y-y') f(x', y') dx' dy' \tag{2}$$

Using Fourier transformation and the convolution theorem, we obtain

$$G(u,v) = H(u,v)F(u,v) \tag{3}$$

where G, H and F are Fourier transform of g, h and f, respectively. This implies that if $H(u,v)$ is known, we can restore $f(x,y)$ by multiplying the Fourier transform $G(u,v)$

of the degraded image by $1/H(u,v)$ and then inverse Fourier transform. This method is usually called inverse filtering.

When noise is present, Fourier transform of Eq.(1), we have

$$G(u,v) = H(u,v)F(u,v) + N(u,v) \tag{4}$$

where $N(u,v)$ is the Fourier transform of $n(x,y)$. The inverse filter H^{-1} gives

$$H^{-1} = \frac{G(u,v) - N(u,v)}{H(u,v)} \tag{5}$$

In the zero neighbourhood of $H(u,v)$, the term of $N(u,v)/H(u,v)$ may have large magnitude. The results of inverse filtering contain many noiselike variations and will not be a meaningful restoration of $f(x,y)$.

One way to avoid arbitrariness in inverse filtering is using Wiener filtering (least squares filtering). On the assumption of the following conditions,

- random field is homogenous
- noise has zero mean
- noise has no correlation with the signal
- the power spectrum density of noise and image is known

we obtain for the restoration filter

$$M_w(u,v) = \frac{H^*(u,v)}{|H(u,v)|^2 + S_n(u,v)/S_f(u,v)} \tag{6}$$

where $M_w(u,v)$ is the transfer function of the Wiener filter that transforms the degraded image into a image which has the least square to the original image. $H^*(u,v)$ is the complex conjugate of transfer function $H(u,v)$, $S_v(u,v)$ and $S_f(u,v)$ are the power spectrum density of noise and image, respectively.

We assumed three different conditions and realized image restoration by inverse filter and Wiener filter. The size of the images are 256×256 in all cases.

2.1 Image degraded by lowpassing Hamming filter

The image is degraded by Hamming filter which can be expressed

$$H_m(u,v) = Rad(u,v)(0.54 + 0.46\cos\frac{\pi\sqrt{u^2+v^2}}{64}) \tag{7}$$

where

$$Rad(u,v) = \begin{cases} 1 & \sqrt{u^2+v^2} \leq 32 \\ 0 & else \end{cases} \tag{8}$$

The degraded image is restored by using the inverse Hamming filter, i.e.,and the transform function $1/H_m(u,v)$.

2.2 Image degraded by moving average

The image is degraded by 16 points moving average in horizontal, and the moving average model is

$$g(x,y) = \frac{1}{16}\sum_{t=0}^{t=15} f(x+t,y) \tag{9}$$

The point spread function $h_{mv}(x,y)$ is achieved by inputing to the system

$$\delta(x,y) = \begin{cases} 1 & x=y=0 \\ 0 & else \end{cases} \tag{10}$$

By performing Fourier transform to the point spread function $h_{mv}(x,y)$, we can get $H_{mv}(u,v)$. The image can be restored by using the inverse filter, i.e.,the transform function $1/H_{mv}(u,v)$.

2.3 Image degraded by lowpassing Hamming filter and random noise

The image is degraded by lowpassing Hamming filter by the same process in case 2.1. Then different magnitute of random noise is added. The signal to noise ratio defined here is the ratio of maximun value of image to the root mean squares of the random noise. The transform function $M_w(u,v)$ of the Wiener filter,

$$M_w(u,v) = \frac{H_m^*(u,v)}{|H_m(u,v)|^2 + S_n(u,v)/S_f(u,v)} \tag{11}$$

where $H_m(u,v)$ is the Hamming filter, $S_v(u,v)$ and $S_f(u,v)$ are the power spectrum density of noise and image, respectively.

In the three cases described in 2.1,2.2 and 2.3, another lowpassing filter is used to avoid possible poles of transform function of restoration before inverse Fourier transform is performed.

3 Results

The original image is the B-mode ultrasound image of a fetus head in the womb shown in Fig.1. The image size is 256 $\times$ 256, and the maxinum grey scale of the image is 255. Figure 2 and 3 represent the image blurred by lowpassing Hamming filter and the image after restoration, respectively. Figure 4 shows the moving average result in horizontal, in which the delicate structures of the image are all lost. Figure 5 is the result of image restoration. Comparing these two images, we can find the image restoration very effective in this case.

Figure 1: The original image of a fetus head in the womb(image size = 256 $\times$ 256).

Figure 2: The image degraded by the lowpassing Hamming filter of 64.

Figure 3: The restored image of Fig.2 by the inverse filter.

Figure 4: The image degraded by 16 point moving average in horizontal.

Figure 5: The restored image of Fig.4 by the inverse filter.

The three images in Fig.6 show the image degraded by lowpassing Hamming filter and additive random noise with different signal to noise ratio of 50, 10 and 2. The results of inverse filter and Wiener filter can be seen in Fig.7 and 8. It can be seen if the noise is present, the inverse filter will be no effect while the Wiener filter is still effective.

The time consumption for inverse and Wiener filtering in the above cases is approximately 8 and 10sec, respectively.

4 Discussion

As we mentioned above, we expect that the image restoration can find its usage in DSE especially when particles velocity in blood vessel is high while the frame rate is relatively low. If the frame rate of the B-mode ultrasound linear scanner is R_f, we have

$$g(x,y) = \int_{-T/2}^{T/2} f(x - m(t), y - n(t))dt \tag{12}$$

where g is the display image of the ultrasound scanner of the moving particles, f is the real image of the moving particles, $m(t)$ and $n(t)$ are the x and y-components of the displacement, respectively, and $T = 1/R_f$. Perform Fourier transform to the above equation, if we assume the motion of the particles interested is uniform in the x- and y-direction with velocity v_x and v_y respectively, that is

$$m(t) = v_x t \tag{13}$$

$$n(t) = v_y t \tag{14}$$

we obtain

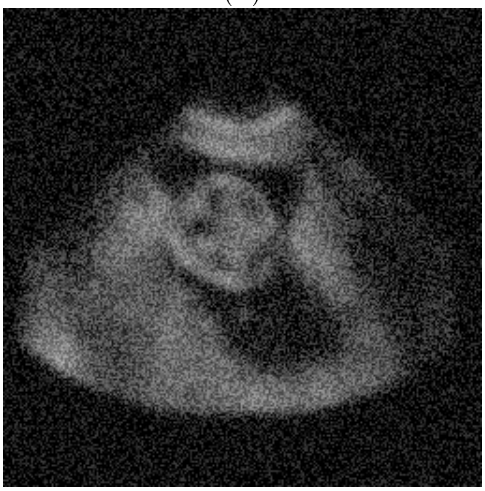
$$H(u,v) = \frac{\sin(\pi(v_x u + v_y v)/R_f)}{\pi(v_x u + v_y v)} \tag{15}$$

where $H(u,v)$ is the transfer function of this B-mode ultrasound image system. If v_x and v_y can be estimated, the blurred image will be restored by the inverse filter.

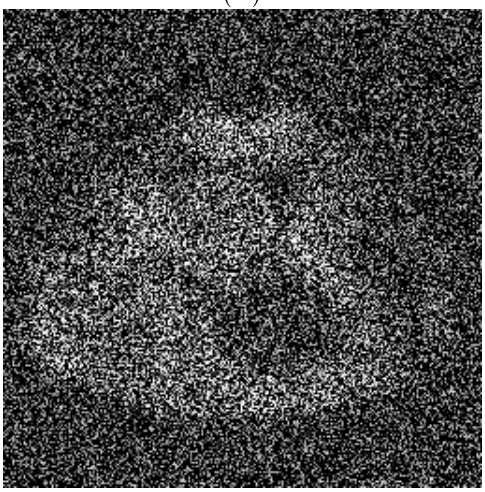
5 Conclusion

The inverse and Wiener filter technique for the image restoration of two-dimensional ultrasound images is discussed. The results of image restoration are also presented. The authors also predict its usage in DSE for the direct vision of blood flow.

(a)



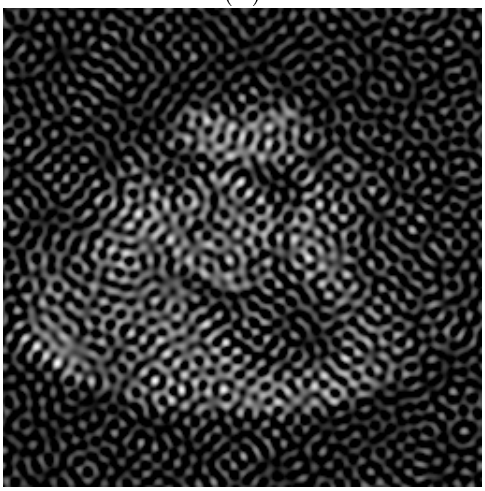
(b)



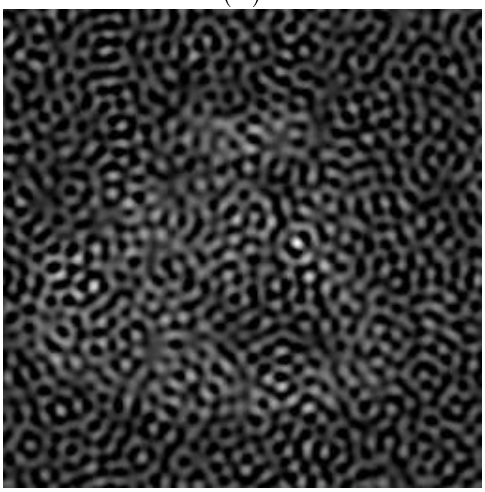
(c)

Figure 6: The images degraded by the lowpassing Hamming filter and noise, (a)S/N = 50 (b)S/N = 10 (c)S/N=2.

(a)



(b)

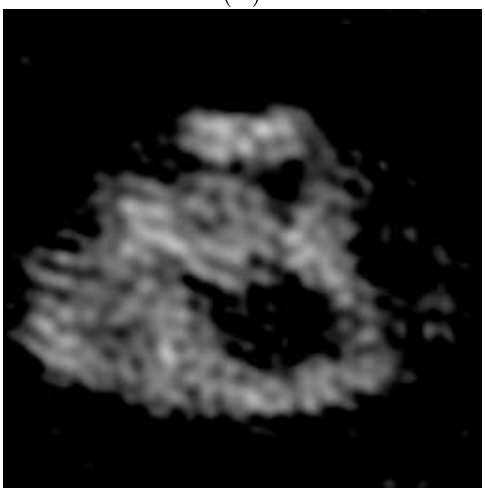


(c)

Figure 7: The restored images of Fig.6 by the inverse filter, (a)S/N = 50 (b)S/N = 10 (c)S/N=2.

(a)

(b)



(c)

Figure 8: The restored images of Fig.6 by the Wiener filter, (a)S/N = 50 (b)S/N = 10 (c)S/N=2.

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