The estimation of the composite Barron functions by multi-layer neural networks in regression problem

Deep learning is known to have high accuracy, but the theoretical reasons for this are not clear. One approach to the theoretical analysis of a statistical method is to restrict the function space to which the true function belongs, and then discuss its convergence rate of the generalization error. A previous study proposed the Barron space and shows that a two-layer neural network can estimate functions in Barron space well in terms of convergence rate. In this study, we define the composite Barron function which is an extension of the Barron space as the true function space, and then theoretically measure the performance of multi-layer neural networks by evaluating the generalization error for the composite Barron function. Specifically, we derive the convergence rate of the generalization error for the composite Barron space by a multi-layer neural network. As a result, we found a multi-layer neural network can achieve a nearly optimal convergence rate in estimating the composite Barron functions. This indicates that a multi-layer neural network is almost optimal as an estimator for the composite Barron space in the minimax sense.