A Polynomial Analysis of Networked Control Systems with Constraints

6th COE Technical Presentation

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Outline in this Presentation

- Introduction
- Networked Control Systems with Constraints
- Polynomial Analysis of Networked Area
- Polynomial Analysis of Constrained Area
- Conclusion

Next Position

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Development of Technology

- Technologies realize our dreams

Increase of the data rate

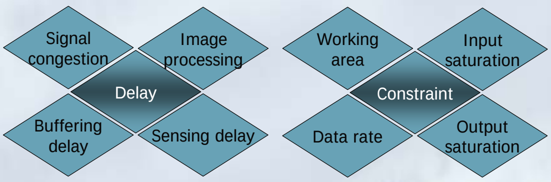
Communication with robots

In the future ...

Problem of Remote Mechanical System

- "Delay" and "constraint" affects the stability

Source of each phenomenon



- Delay occurs mainly in the networked area
- Constraint exists mainly in the mechanical area

Deficiency of Traditional Approach

- Rough analyses produce conservative results

In the constrained area

Conventional (LMI) approach

Level set of V(x) ⇨ Reachable set ⇨ Constraint is satisfied

Use of quadratic form $V(x) = x^T P x$

In the networked area

Conventional (robust control) approach

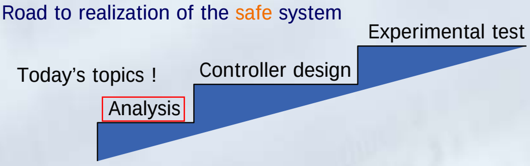
Approximated by

Nominal delay + uncertainty

Vision of this Research

- Exact "analysis" and "controller design" of networked control systems with constraints

Road to realization of the safe system



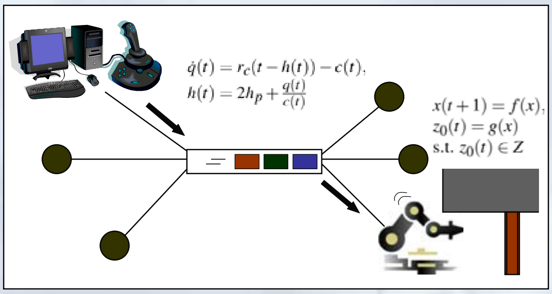
Following outline

1. Problem statement about the system
2. Polynomial analysis of networked area
3. Polynomial analysis of constrained area

Next Position

Problem Statement

- Two types of model exist in this system



Next Position

Main Idea of Networked Area

- Introduction of the pade approximation and the SOS

Network congestion model

$$\dot{q}(t) = r_c(t-h(t)) - c(t),$$
$$h(t) = 2h_p + \frac{q(t)}{c(t)}$$

Transfer func. form

$$sq(s) = e^{-Ls} r_c(s) - \frac{c(s)}{s},$$

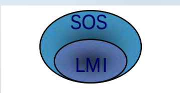
Pade approximation (1st order)

$$sq(s) = \frac{1-Ls/2}{1+Ls/2} r_c(s) - \frac{c(s)}{s},$$

State space form

$$\dot{x}(t) = A(q)x(t) + Bu(t),$$
$$q(t) = C(q)x(t) + D,$$
$$L := 2h_p + \frac{q(t)}{c(t)}$$

SOS analysis (polynomial)



State dependent form is handled as polynomial form

Simulation Result

- Simple examples show the effectiveness

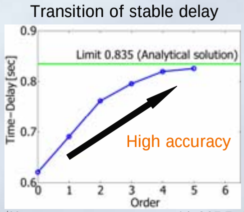
Model 1 :

$$\dot{x}(t) = -1.3x(t-r) + x(t)$$

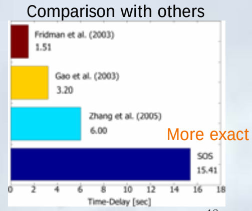
Model 2 :

$$\dot{x}(t) = Ax(t) + A_1x(t-r),$$
$$A = \begin{bmatrix} 0 & 0 \\ -70.18 & -76.67 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}$$

Transition of stable delay



Comparison with others



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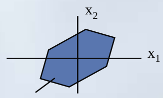
Main Idea of Constrained Area

- Introduction of the MOA set and the SOS

For linear system

$$x(t+1) = Ax(t),\quad z_0(t) = C_0 x(t)\quad \text{s.t. } z_0(t) \in Z$$

LMI →



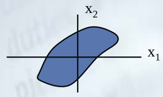
Is the constraint satisfied ?

MOA set
(Set of initial states
satisfying the constraint)

For polynomial system

$$x(t+1) = f(x),\quad z_0(t) = g(x)\quad \text{s.t. } z_0(t) \in Z$$

SOS →



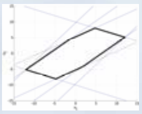
Simulation Result

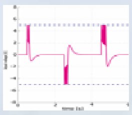
- Anticipation of the effectiveness

MOA set | Time response

Linear system :

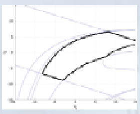
$$x(t+1) = Ax(t),\quad z_0(t) = C_0 x(t)\quad \text{s.t. } z_0(t) \in Z$$





Polynomial system :

$$x(t+1) = f(x),\quad z_0(t) = g(x)\quad \text{s.t. } z_0(t) \in Z$$



Simulating ..

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Conclusion

- A polynomial analysis of networked control systems with constraints :
  - A system can be handled exactly more than others
  - A wider class of systems can be treated
- Future work :
  - Analysis of higher order approximated systems
  - Expansion of the switching controller to polynomial ver.
  - Connection of each areas