Sum of Squares Condition for Stability of Time-Delay Systems

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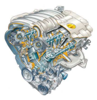
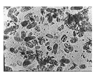
Outline

- Motivation
- Time-delay System
- Sum of Squares (SOS)
- Numerical Example
- Summary and Future Plan

Motivation

There are time-delay systems in many different areas.

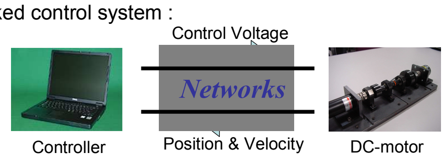

IC Engine


Biology


Network


Chemical reaction

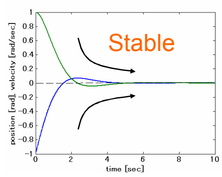
Networked control system :



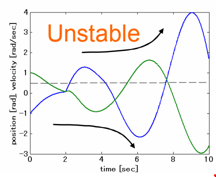
Motivation

The time-delay frequently causes instability and performance degradation.

Time responses of the networked control system :



No delay



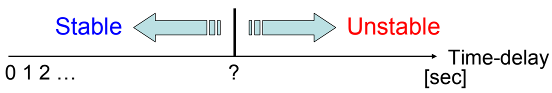
Delay 2 [sec]

Difficult issue

Motivation

Stability analysis for time-delay systems has been one of the most challenging issues.

In particular …
Important to judge if the time-delay system works 'safely' .

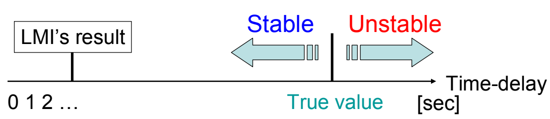


Problem :
How do we find this borderline ?

Motivation

In the conventional method, Linear Matrix Inequality (LMI) based on linear calculation is used.

However …
LMI approach can find only a conservative borderline.



Conservative !!

Motivation

Our goal :

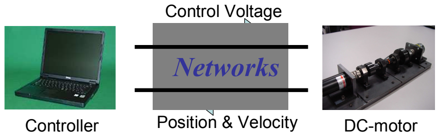
- Show an another analysis method to be less conservative.

To achieve such the goal :

- Introduce Sum of Squares (SOS) methodology instead of LMI.
- Conduct a numerical check .

Time-delay system

This presentation considers this simplest model as a time-delay system.



$$\dot{x}(t) = Ax(t) + A_1 x(t-\tau) + Bu(t)$$

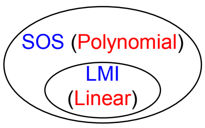
Information delay

(Position data, velocity data, ･･･)

Sum of Squares

To derive a less conservative condition, SOS, being analysis tool based on polynomial, is introduced.

Feature of SOS :



SOS includes LMI.

Then…
SOS is better than LMI.

⇨ A less conservative condition.

{Details are omitted, because mathematical knowledge is required.}

SOS condition

Time-delay system :

$$\dot{x}(t) = Ax(t) + A_1 x(t-\tau) + Bu(t)$$

SOS condition to judge the stability :

The time-delay system is stable , if there exists a positive constant $\varepsilon > 0$ and polynomials $a_0(x),\ a_1(\theta,\xi,x,y,z),\ a_2(x_t(\zeta))$ and $a_3(x_t(\zeta))$ such that the following condition hold :

C1 : $a_0(x) - \varepsilon||x||^2$ is SOS,

C2 : $a_1(\theta,\xi,x,y,z) + \sum_{j=1}^{2} q_{1,j} g_i(\theta,\xi)$ is SOS

C3 : $a_2(x_t(\zeta))$ is SOS, $a_3(x_t(\zeta))$ is SOS,

C4 : $\frac{da_0}{dx} f + \tau^2 (\frac{da_1}{dx} f - \tau^2 \frac{da_1}{d\theta} - \tau^2 \frac{da_1}{d\xi}) + \tau a_2(x) - \tau a_2(y)$
$+ \tau a_3(x) - \tau a_3(z) + \tau a_1(0,\xi,x,x,z) - \tau a_1(-\tau,\xi,x,x_d,z)$
$+ \tau a_1(\theta,0,x,y,x) - \tau a_1(\theta,-r,x,y,x_d) - \varepsilon||x||^2 + \sum_{j=1}^{2} q_{2,j} g_i(\theta,\xi)$ is SOS

Remark : $x_d = x_t(-\tau),\ y = x_t(\theta),\ z = x_t(\xi),\ x = x_t(-\tau)$

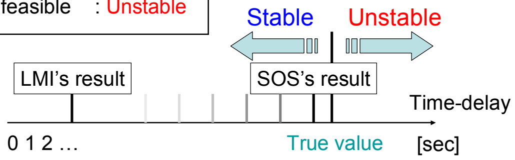
SOS condition

Find the maximum delay within the system being stable.

Given : Delay τ

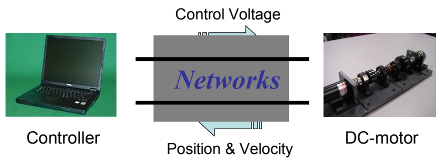
If SOS condition is
  Feasible : Stable
  Infeasible : Unstable

Iterative procedure !



Numerical Example

Consider the following simple time-delay system :



$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}\begin{bmatrix} x_1(t-\tau) \\ x_2(t-\tau) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

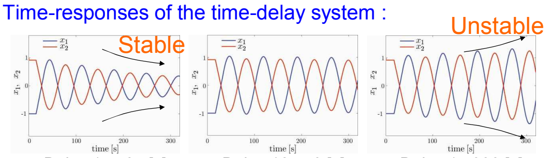
$$u(t) = \begin{bmatrix} -70.18 & -77.67 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Numerical Example

Calculation results by using SOS and LMI condition :

Method	LMI	SOS	True value
Max Delay [s]	6.00	15.407	16.770

Time-responses of the time-delay system :



Delay 15.407 [s] Delay 16.770 [s] Delay 17.300 [s]

SOS condition provides better results than LMI approach.

Summary

Summary :

- I showed SOS condition for the stability of the time delay system.
- I performed the stability check, illustrating time-responses of a example system .
- I specified that SOS condition is better than LMI condition .

Future plan :

- Controller design for the time-delay system by SOS approach.
- Stability analysis for the system with constraints .
- Experiment of the technique.

Thank you for your attention