Model Predictive Control for a Nonlinear Hybrid Dynamical System

D1 Yoshiyasu SAKAKURA
(Systems and Control Lab.)

Outline

- Motivation
  - What a hybrid (dynamical) system is
  - COE program and hybrid systems
- MPC with MLDS model
  - An explanation about the method used in my study
- Case Study Plant
  - The temperature control problem of CSTR
- Control Results
  - Traditional MPC
  - MPC with a human idea
- Summary and Future works

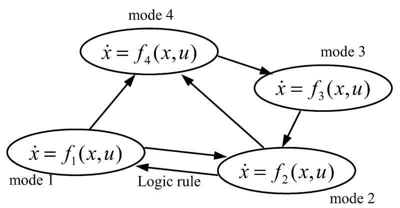
Motivation

- What is a hybrid dynamical system (hybrid system)?

integrated continuous dynamics and discrete events

Examples
- Human behavior
- Communication systems
- Mechanical systems
- Industrial systems

:



There are multiple modes and logic rules in a hybrid dynamical system.

- Each mode is described as DAEs or difference Eqs. – continuous dynamics
- Logic rules associate each mode. – discrete events

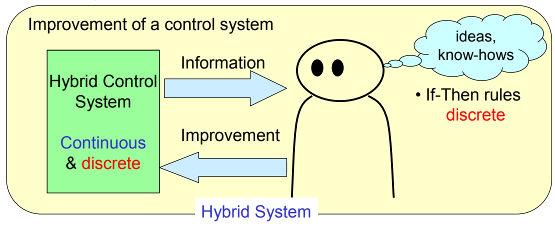
Motivation

- COE program and hybrid systems

one of the research objects

The man-machine systems also can be seen as hybrid systems.

In this study



The hybrid control approach provides an user-friendly control scheme.

Motivation

Approaches for a hybrid system

- Switching control
- Multiple model approach
- MPC with MLDS model
  (MPC : Model Predictive Control, MLDS : Mixed Logical Dynamical System)

In this presentation

- Propose the method to apply human ideas to a control system.
  - I consider an improvement problem of a control system as a hybrid system.
  - MLDS formulation is used to handle a hybrid system in a control.
- Show a concrete example
  - A temperature control problem of a CSTR (nonlinear system) is considered.
  - Our control scheme provides superior cost performance.

MPC with MLDS model

MPC with MLDS model method consist of 2 techniques.

- MPC scheme (described as an optimization problem)
- MLDS formulation (transform discrete events into inequalities)

Model Predictive Control

The MPC problem is described as a constrained optimization problem as follows;

$$\underset{u(k,\cdots,k+i)}{\mathrm{Min}} \; J(k) = \sum_{j=H_w}^{H_p} \|y(k+j) - r(k+j)\|_Q^2 + \sum_{i=0}^{H_u-1} \|\Delta u(k+i)\|_R^2$$

$$\Delta u(k+i) = u(k+i) - u(k+i-1)$$

subject to:

$$\frac{dx}{dt} = f(x,u), \quad y = h(x)$$ process model (differential Eqs. / difference Eqs.)

$$u_{\min} \le u_{k+j} \le u_{\max}$$ constraints (inequalities)

If the discrete events are transformed into inequalities, hybrid control are realized.

MPC with MLDS model

MLDS Formulation

- The method to transform If-then rules in a hybrid system into inequalities

Example

condition expression (discrete event)

$$\begin{cases} x(k+1) = Ax(k) + B_1 u(k) & \text{if } m \le Cx(k) < 0 \quad \leftrightarrow \delta(k) = 0 \\ x(k+1) = Ax(k) + B_2 u(k) & \text{if } 0 \le Cx(k) \le M \quad \leftrightarrow \delta(k) = 1 \end{cases}$$

Introduce a binary parameter $\delta(k)$ corresponding to condition expression

$$x(k+1) = Ax(k) + (B_1 + (B_2 - B_1)\delta(k))u(k)$$

$$-m\delta(k) + m \le Cx(k) \le (M+\varepsilon)\delta(k) - \varepsilon$$

where

$\delta(k) = \{0, 1\}$

ε : small positive scaler

MPC with MLDS model

$$\begin{cases} \text{if } m \le Cx(k) < 0 & \leftrightarrow \delta(k) = 0 \\ \text{if } 0 \le Cx(k) \le M & \leftrightarrow \delta(k) = 1 \end{cases}$$

$$-m\delta(k) + m \le Cx(k) \le (M+\varepsilon)\delta(k) - \varepsilon$$

If the case $\delta(k) = 0$ $([\delta(k) = 0] \leftrightarrow [Cx(k) < 0])$

$$-m\delta(k) + m \le Cx(k) \le (M+\varepsilon)\delta(k) - \varepsilon$$

$$m \le Cx(k) < 0$$

If the case $\delta(k) = 1$ $([\delta(k) = 1] \leftrightarrow [0 \le Cx(k)])$

$$-m\delta(k) + m \le Cx(k) \le (M+\varepsilon)\delta(k) - \varepsilon$$

$$0 \le Cx(k) \le M$$

MPC with MLDS model

The MPC problem for the hybrid system is described as MIQP.

Mixed Integer Quadratic Problem

$$\underset{u(k,\cdots,k+i)}{\text{Min}} J(k) = \sum_{j=H_w}^{H_p} \|y(k+j) - r(k+j)\|_Q^2 + \sum_{i=0}^{H_u-1} \|\Delta u(k+i)\|_R^2$$

$$\Delta u(k+i) = u(k+i) - u(k+i-1)$$

subject to:

$$\frac{dx}{dt} = f(x, u)$$

$$y = f(x)$$

$$-m\delta(k) + m \le Cx(k) \le (M+\varepsilon)\delta(k) - \varepsilon$$

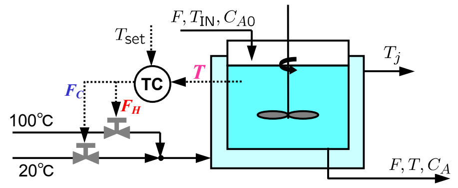
$$u_{\text{max}} \le u(k) \le u_{\text{min}}$$

where $\delta(k) = \{0, 1\}$

ε : small positive scaler

Case Study Plant

- Temperature control problem of a CSTR (Continuous Stirred Tank Reactor)



- Continuous dynamics
- Highly nonlinear reaction kinetics
  - Liquid phase exothermal reaction occurs in the reactor.

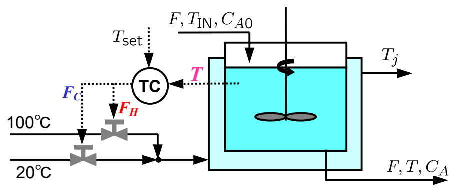
T : controlled variable (reactor temperature)

F_C, F_H : manipulated variables (flow rates of cold water and hot water)

Nonlinear 2 inputs 1 output system

Case Study Plant

- Mathematical model of the CSTR



$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A0} - C_A) - kC_A$$

$$\frac{dT}{dt} = \frac{F}{V}(T_{\text{IN}} - T) + \frac{k(-\Delta H)C_A}{\rho C_p} - \frac{UA_h}{\rho V C_p}(T - T_j)$$

$$\frac{dT_j}{dt} = \frac{T_H - T_j}{V_j}F_H + \frac{T_C - T_j}{V_j}F_C + \frac{UAh}{\rho_j V_j C_{pj}}(T - T_j)$$

$$k = k_0 \exp\left(\frac{-E}{RT}\right)$$

highly nonlinear

rate coefficient of reaction

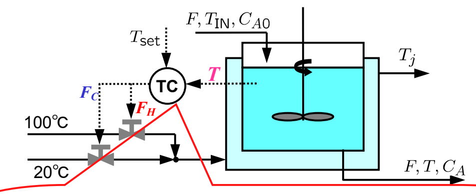
constraints

$0 \le F_C \le 3$

$0 \le F_H \le 3$

Control Results

- Evaluation function of Traditional MPC



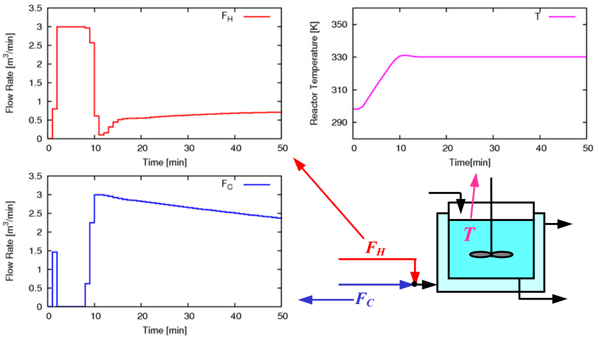
$$J(k) = \sum_{j=1}^{5} \left\{ \|T(k+j) - T_{\text{set}}(k+j)\|_Q^2 + \|\Delta U(k+j-1)\|_R^2 + \|U(k+j-1)\|_S^2 \right\}$$

$$U = (F_C, F_H), \quad \Delta U(k+j-1) = U(k+j-1) - U(k+j-2)$$

$Q = 10$, $R, S =$ tuning parameter vectors

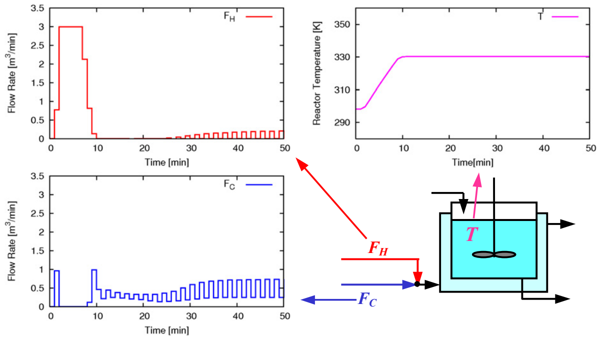
This traditional MPC used in a control of CSTR temperature in a case study.

Control Results Case1: Traditional MPC R=(1,1) S = (0.01, 0.01)



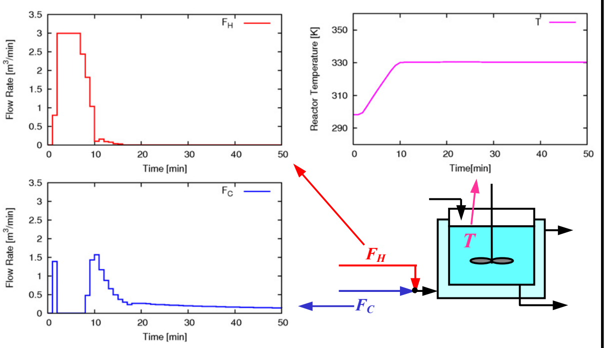
Good control, but not efficient (too much Hot and Cold water is used)

Control Results Case2: TMPC (after several trials) R=(1,1) ,S = (1,1)



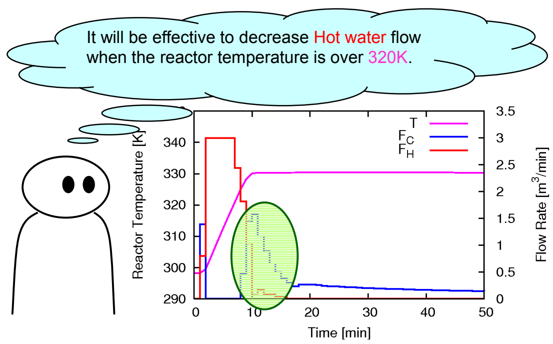
Cost performance becomes better, but inputs are oscillated.

Control Results Case3: TMPC (after much trial and error)R=(10,10) ,S = (1,1)



It seems tough to fined parameter vectors to achieve more efficient control.

Idea for an effective control



Plot of the inputs and output on a graph

Idea for an effective control

It will be effective to decrease Hot water flow when the reactor temperature is over 320K.

if $m \le T(k) < 320\text{K} \quad \leftrightarrow \quad [\delta(k) = 0]$

then $J(k) = \sum_{j=1}^{5} \left\{ \|T(k+j) - T_{\text{set}}(k+j)\|_{10}^2 + \|\Delta U(k+j-1)\|_{(10,10)}^2 + \|U(k+j-1)\|_{(1,1)}^2 \right\}$

if $320\text{K} \le T(k) \le M \quad \leftrightarrow \quad [\delta(k) = 1]$

then $J(k) = \sum_{j=1}^{5} \left\{ \|T(k+j) - T_{\text{set}}(k+j)\|_{10}^2 + \|\Delta U(k+j-1)\|_{(10,10)}^2 + \|U(k+j-1)\|_{(1,10)}^2 \right\}$

$U = (F_C, F_H), \quad \Delta U(k+j-1) = U(k+j-1) - U(k+j-2)$

The control system with the human idea is described as a hybrid system.

MLDS Formulation

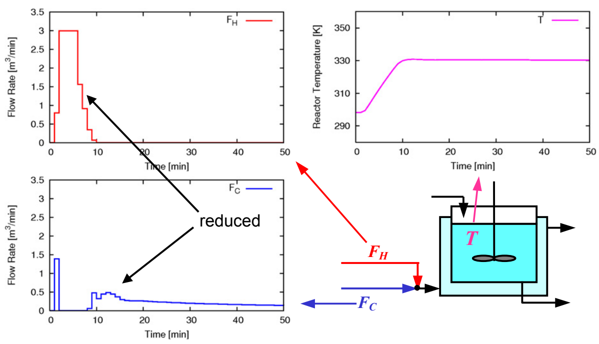
$$J(k) = \sum_{j=1}^{5} \left\{ \|T(k+j) - T_{\text{set}}(k+j)\|_{10}^2 + \|\Delta U(k+j-1)\|_{(10,10)}^2 + \|U(k+j-1)\|_{(1,1+10\delta(k))}^2 \right\}$$

$$-m\delta(k) + m + 320 \le T(k) \le (M+\varepsilon)\delta(k) - \varepsilon + 320$$

$$\delta(k) = \{0, 1\}$$

The Human idea can be apply to control system by using MLD formulation.

Control Results Case4: MPC with a human idea



As Inputs are reduced, the human idea woks well in this case study.
Our control scheme provides superior cost performance.

Summary and Future Works

summary

- Propose the method to apply human idea to a control system.
  - I considered a man-machine system as a hybrid system.
  - MLDS formulation was used to handle discrete events in a hybrid system.
- Show a simulation example
  - The temperature control problem of a CSTR was considered.
  - Our control scheme provided superior cost performance in a control.

Future works

- Address the fuzzy expression of a human
- Fault tolerant control system

Thank you for your attention