A Reference Governor Control Approach to Closed-loop Systems with State and/or Control Constraints

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Nara Institute of Science and Technology

Kiminao KOGISO
Systems and Control Laboratory

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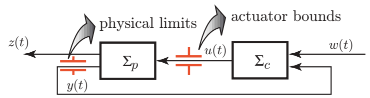
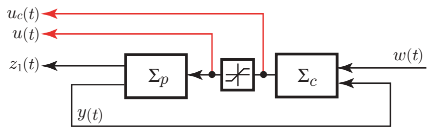
Constrained system

- Closed-loop systems with state and/or control constraints.
- A real control system necessarily has these constraints.



- If such constraints are violated,
  - degrade a system performance,
  - lead to instability of the designed control system.

Numerical example of constraint violation



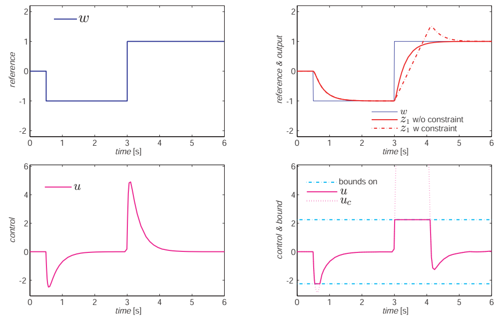
- Plant Σ_p and controller Σ_c in state-space description

$$\Sigma_p : \begin{bmatrix} \dot{x}_p \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_p \\ u \end{bmatrix}, \quad \Sigma_c : \begin{bmatrix} \dot{x}_c \\ u_c \end{bmatrix} = \begin{bmatrix} 0 & 3 & -3 \\ 30 & 0 & -30 \end{bmatrix} \begin{bmatrix} x_c \\ w \\ z_1 \end{bmatrix}$$

- Constraint condition about control input

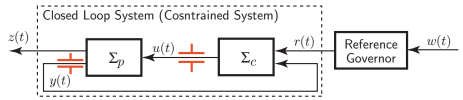
$$-2.25 \le u \le 2.25$$

Numerical example of constraint violation



Control output w/o constraint — Control output w/ constraint

Reference governor(RG) approach



- Manage a time-varying reference w into r to fufill the constraint.
  - Kapasouris and others, 1988. Hirata and Fujita, 1999. Gilbert and Kolmanovsky, 2002. Oh-hara and hirata, 2002
- Consider a tracking performance under a constant reference $\bar{w}$. (Suitable for mechanical systems)
  - Sugie and Yamamoto, 2001. Kogiso and Hirata, 2003.
- A few reports on experimental validation of the effectiveness.

Goals in this study

- Constructing a reference governor for improvement of control performance by achieving constraint fulfillment
- Experimental validation of the proposed method

Improvements in this study

- A feasibility of a reference management (optimization) problem
- A reference governor in a piecewise state affine function over the specified state region(i.e., $r = Fx + g \quad \forall x \in CR$).

  We easily expect that the proposing method can be more robust for noises and unmodelled dynamics than the previous work...

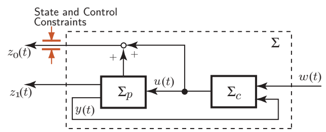
- A validation of the effectiveness using a real control system.

Formulation of constrained system (tracking ctrl.)

- Linear discrete-time system: Σ

  $x = [x_p' \ x_c']' \in \Re^n,\ x(0) = x_0$

$$x(t+1) = Ax(t) + Bw(t)$$
$$z_1(t) = C_1 x(t)$$
$$z_0(t) = C_0 x(t) + D_0 w(t)$$



- Pointwise-in-time constraint condition:

$$z_0(t) \in \mathcal{Z} := \{z_0 \in \Re^{p_0} \mid M_z z_0 \le m_z\} \quad \forall t \in \mathbb{Z}^+$$

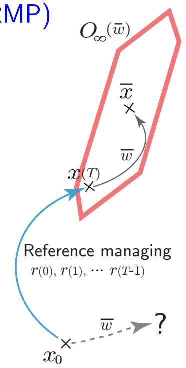
- Assumptions:

  A full state x of Σ is measurable, and a reference is constant $\bar{w} \in \mathrm{int}W$.

Reference management problem(RMP)

$$\min_{\hat{r}^T} ||\hat{z}_1^T - \hat{\bar{w}}^T||_{2,P} + ||\hat{r}^T - \hat{\bar{w}}^T||_{2,Q}$$
$$\text{s.t.}\ \ z_0(\tau) \in \mathcal{Z}, \quad \tau = 0, 1, \cdots, \infty$$

$\hat{r}^T = [r(0)' \cdots r(T-1)']',\ \hat{z}_1^T = Q_1 x_0 + Q_2 \hat{r}^T,$
$||z||_{2,P} = z'Pz,\ P = P' \succ 0$



Reference management problem(RMP)

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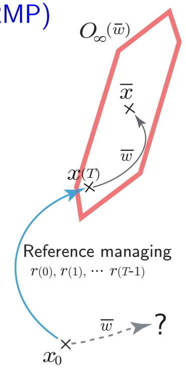
$$\text{s.t.}\ \ z_0(\tau) \in \mathcal{Z}, \quad \tau = 0, \cdots, T-1$$
$$x(T) \in O_\infty(\bar{w})$$

$\hat{r}^T = [r(0)' \cdots r(T-1)']',\ \hat{z}_1^T = Q_1 x_0 + Q_2 \hat{r}^T,$
$||z||_{2,P} = z'Pz,\ P = P' \succ 0$

Maximal Output Admissible set: $O_\infty(\bar{w})$

MOA set for Σ has the following properties:
i) $x(T) \in O_\infty(\bar{w}) \Leftrightarrow z_0(\tau) \in \mathcal{Z},\ \forall \tau \ge T,$
ii) a convex polyhedral set.



RG in a piecewise state affine fucntion

1. RMP(is feasible for given T^*):

$$\min_{\hat{r}^{T^*}} ||\hat{z}_1^{T^*} - \hat{\bar{w}}^{T^*}||_{2,P} + ||\hat{r}^{T^*} - \hat{\bar{w}}^{T^*}||_{2,Q}$$
$$\text{s.t.}\ \ x(T^*) \in O_\infty(\bar{w})$$
$$z_0(\tau) \in \mathcal{Z} \quad \tau = 0, \cdots, T^* - 1$$

2. Multi-parametric quadratic programming problem(also feasible):
   a variable $z = \hat{r}^{T^*} + H^{-1} f(x) \in \Re^{p_1 T^*}$, a state parameter $x \in \Re^n$,

$$V_z(x) = \min_z z'Hz \quad \text{s.t.}\ \ Gz \le N + Sx$$

3. Using KKT condition of MpQP, calculate F^{T^*} and g^{T^*} that gives the optimal solutoin $\hat{r}^{T^*}$ to RMP for a measured state $x \in CR^{T^*}$.

$$\hat{r}^{T^*} = F^{T^*} x + g^{T^*} \quad \forall x \in CR^{T^*}$$

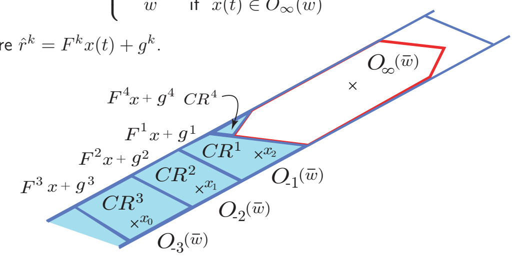
2. → 3. Bemporad's explicit LQR, *Automatica* 2002

RG in a piecewise state affine fucntion

Reference management rule:

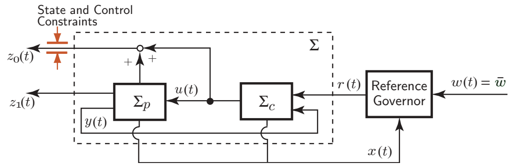
$$r(t) = \begin{cases} \hat{r}^k(1) & \text{if}\ \ x(t) \in CR^k,\ \ k = 1, \cdots, T^* \\ \bar{w} & \text{if}\ \ x(t) \in O_\infty(\bar{w}) \end{cases}$$

where $\hat{r}^k = F^k x(t) + g^k$.



Reference Governor

RG in a piecewise state affine fucntion

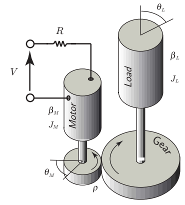


The constrained system equipped with RG

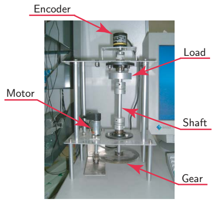
For a measured state $x(t)$, RG manages the constant reference $\bar{w}$ into r and achieves the fulfillment of the constraint, $z_0(t) \in \mathcal{Z},\ \forall t \in \mathbb{Z}^+$, of Σ. Furthermore, the output z_1 has a good tracking performance.

Simulation & experimental result

DC position servomechanism



(a) A model illustration



(b) A plant picture

- Control $u = V$, output $z_1 = \theta_L$, and constraint $|u| \leq 2.4$ [V]
- Control object:
  Tracking control of θ_L to a reference $\bar{w}$ fulfilling the constraint

Simulation & experimental result

Model of DC position servomechanism

$$\frac{\theta_L(s)}{V(s)} = \frac{w_n^2}{s(s + 2\zeta w_n)}$$

- Identification: $\zeta = 0.7,\ w_n = 7.0$
  (by data of 90 [deg] step response)

Design and implementation of PI-controller

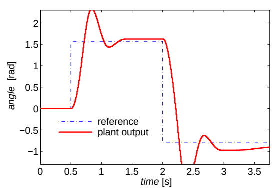
$$K_c\left(1 + \frac{1}{T_c s}\right)$$

- Stabilization and tracking: $K_c = 3,\ T_c = 3$
- Discretized by sampling time 10 [ms] with a zero-holder
- Implemented to a computer (RT-Linux v3.1 Interl 3 733MHz 256MB)

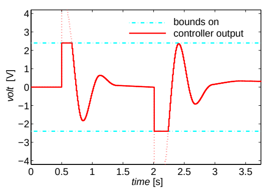
Simulation & experimental result

Experimental result without RG

$$\text{Reference } w(t) = \begin{cases} 0, & 0 \leq t < 0.5 \text{ [s]}, \\ \bar{w}_1, & 0.5 \leq t < 2.0 \text{ [s]}, \\ \bar{w}_2, & 2.0 \leq t \text{ [s]}, \end{cases} \qquad \begin{aligned} \bar{w}_1 &= 1.5708 \text{ [rad]} \\ \bar{w}_2 &= -0.7853 \text{ [rad]} \end{aligned}$$



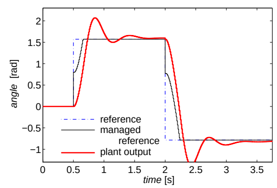
(a) Reference w and controlled output θ_L



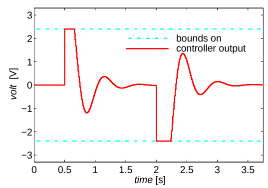
(b) Controller output u and the constraint $|u| \leq 2.4$ [V]

Simulation & experimental result

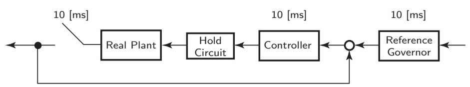
Simulation result with RG



(a) Managed reference r and controlled output θ_L

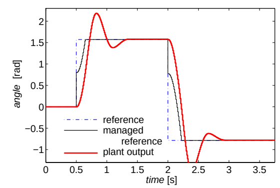


(b) Controller output u and the constraint $|u| \leq 2.4$ [V]



Simulation & experimental result

Experimental result with RG



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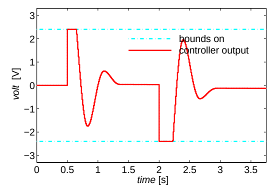
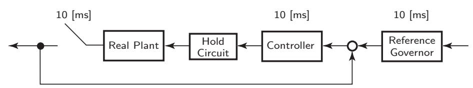


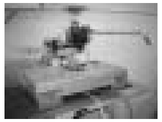
(b) Controller output u and the constraint $|u| \leq 2.4$ [V]



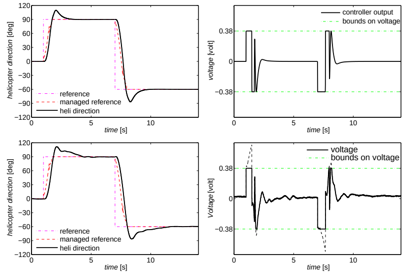
Summary

- We have constructed a reference governor in a state affine function which is derived from the reference management (optimization) problem.
- We have clarified the state region, which gives a feasiblity to the reference management problem.
- To validate the effectiveness, we have performed a simulation and experiment. This experimental result has neither conservatism nor violation for the constriaint fulfillment at all, and demonstrates the validity of this proposed method under a modeling error.

Experimental results by the most fundamental RG[a]



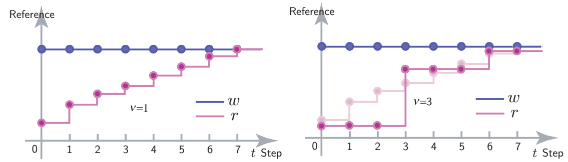
RC helicopter:
Control of yaw angle in hovering.



[a] "A performance improving off-line reference management for systems with state and control constraints", IEEE CDC 2001

Another study

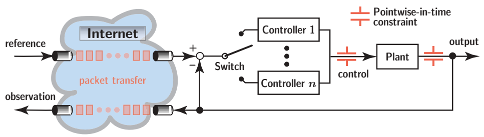
- Multi-sampled implementation of RG



  - Sampling time of Σ: T_s
  - Reference management period: νT_s
  - Constraint fulfillment for each T_s sampled data
  - Reduction of implemented data of RG

Future plan

- Control approach to constrained systems which includes networks.



  - Dynamical control approach to networks
  - Simulation and experimental validation of effectiveness and practicability
- Control applications for educational goals over networks